MODELING AND SIMULATION OF MICROELECTRONIC DEVICES

SKEL 4653

TOPIC 4 : CHARGE TRANSPORT

BASED ON THE BOOK: NANOELECTRONICS - QUANTUM ENGINEERING OF LOW-DIMENSIONAL NANOENSEMBLES

Modeling and Simulation of Microelectronic Devices

Topic 4: Charge Transport

Lecture 1

Drift Response to Electric Field Theory

$$v_{Dd} = v_{satd} \tanh\left(\frac{\mathcal{E}}{\mathcal{E}_{cd}}\right)$$

$$\mathcal{E}_{cd} = \frac{v_{satd}}{\mu_{o\infty d}}$$

$$\mu_{o\infty Deg} = \frac{q\ell_{o\infty} v_{id}}{dk_B T_{od}}$$

$$v_{satd} = v_{ud} \tanh(\delta_Q)$$

$$T_{od} = T \frac{\Im_j(\eta)}{\Im_{j-1}(\eta)}$$

Now onward we will drop d which is redundant for given dimensionality

Drift Response to Electric Field Theory

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Drift Response to Electric field
Theory and Empirical

Theory : $v_D = v_{sat} \tanh\left(\dfrac{\mathcal{E}}{\mathcal{E}_c}\right)$

Empirical :

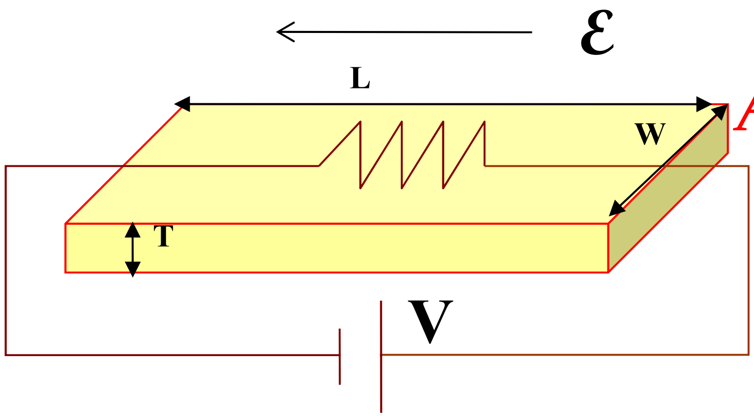
$$v_D = v_{sat} \frac{\dfrac{\mathcal{E}}{\mathcal{E}_c}}{\left[1+\left(\dfrac{\mathcal{E}}{\mathcal{E}_c}\right)^{\gamma}\right]^{\frac{1}{\gamma}}}$$

$\gamma = 2.8$ gives perefect fit to the theory

Transport Defined (3D)

Transportation of charge over the length of the channel

$$I_n = \frac{Q_n}{t} = \frac{n_3(LWT)(-q)}{t}$$



$$A_c = WT$$

$$I_n = -n_3\frac{L}{t}qA_c = -n_3 v_{Dn} qA_c$$

$$I_p = \frac{Q_p}{t} = \frac{p_3(LWT)(+q)}{t}$$

$$I_p = +p_3\frac{L}{t}qA_c = +p_3 v_{Dp} qA_c$$

Ohm's Law (20th Century Paradigm)
Linear Drift Response to the Electric Field

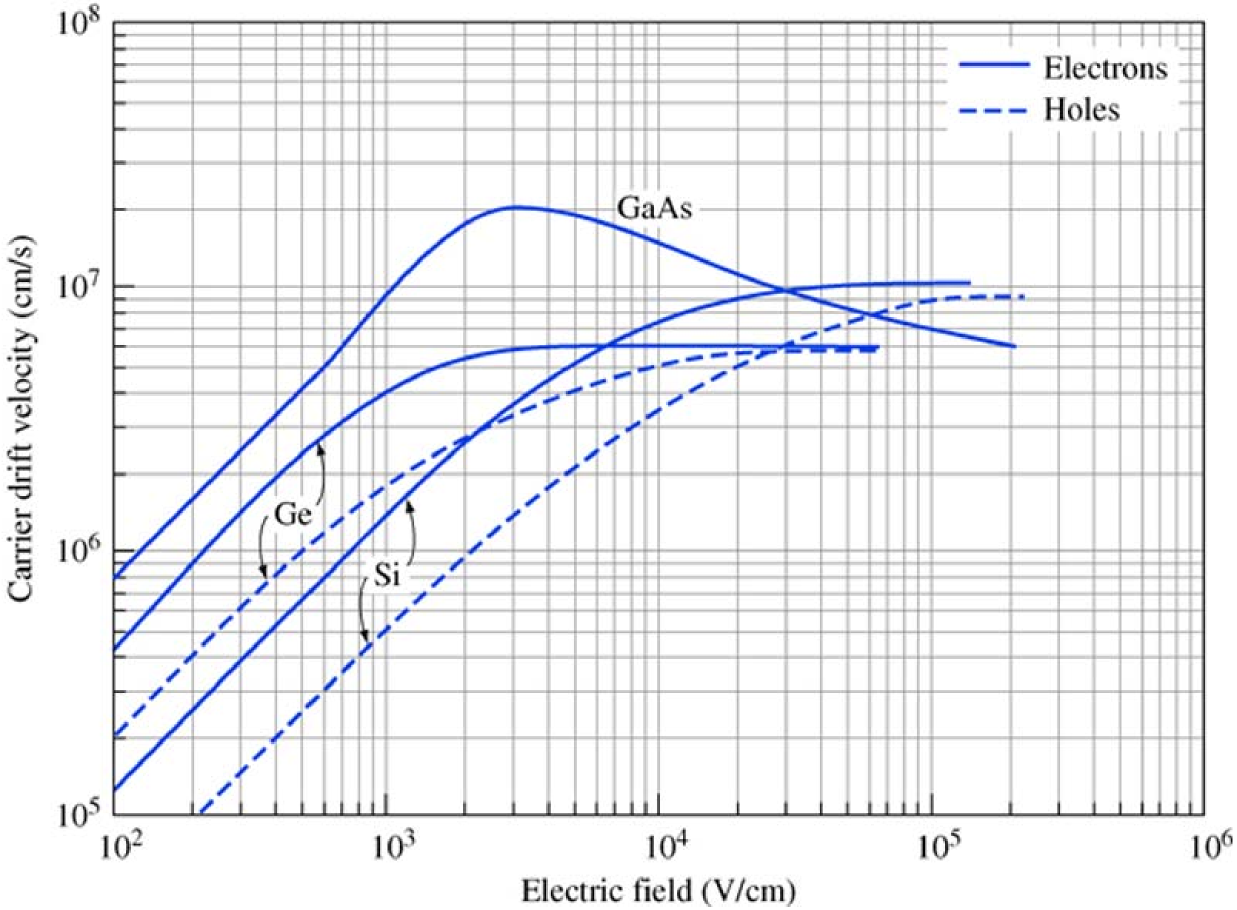
$$v_{Dn} = -\mu_{on}\mathcal{E} \qquad v_{Dp} = +\mu_{op}\mathcal{E} \qquad \mathcal{E} = \frac{V}{L}$$

$$I = I_n + I_p = -n_3(-\mu_{on}\mathcal{E})qA_c + p_3(+\mu_{op}\mathcal{E})qA_c$$

$$I = \left(n_3\mu_{on} + p_3\mu_{op}\right)\frac{V}{L}qA_c \equiv \frac{V}{R_o} \qquad R_o = \rho_3\frac{L}{A_c}$$

$$\rho_3 = \frac{1}{\sigma_3} = \frac{1}{\left(n_3\mu_{on} + p_3\mu_{op}\right)q} \text{(unit} : \Omega \cdot m)$$

What Experiments Show?



Saturation Current

$$v_D = -v_{sat}$$ *Electrons*

$$v_D = +v_{satp}$$ *Holes*

$$I_{satn} = n_3 v_{sat} q A_c$$

$$I_{satp} = p_3 v_{sat} q A_c$$

Saturation current does not depend on length!!!

Sat Law (21st Century Paradigm)

Nonlinear Drift Response to the Electric Field

$$\textit{Theory}: \; I_n = n_3 q v_D A_c = n_3 q v_{sat} A_c \tanh\left(\frac{\mathcal{E}}{\mathcal{E}_c}\right)$$

$$I_n = I_{satn} \tanh\left(\frac{V}{V_c}\right) = \frac{V}{R_o} \frac{\tanh\left(\frac{V}{V_c}\right)}{\frac{V}{V_c}} \qquad \mathcal{E} = \frac{V}{L} \quad \mathcal{E}_c = \frac{V_c}{L}$$

$$I_n = \begin{cases} \dfrac{V}{R_o} & V << V_c \\ I_{sat} & V >> V_c \end{cases} \qquad R_o = \frac{V_c}{I_{sat}}$$

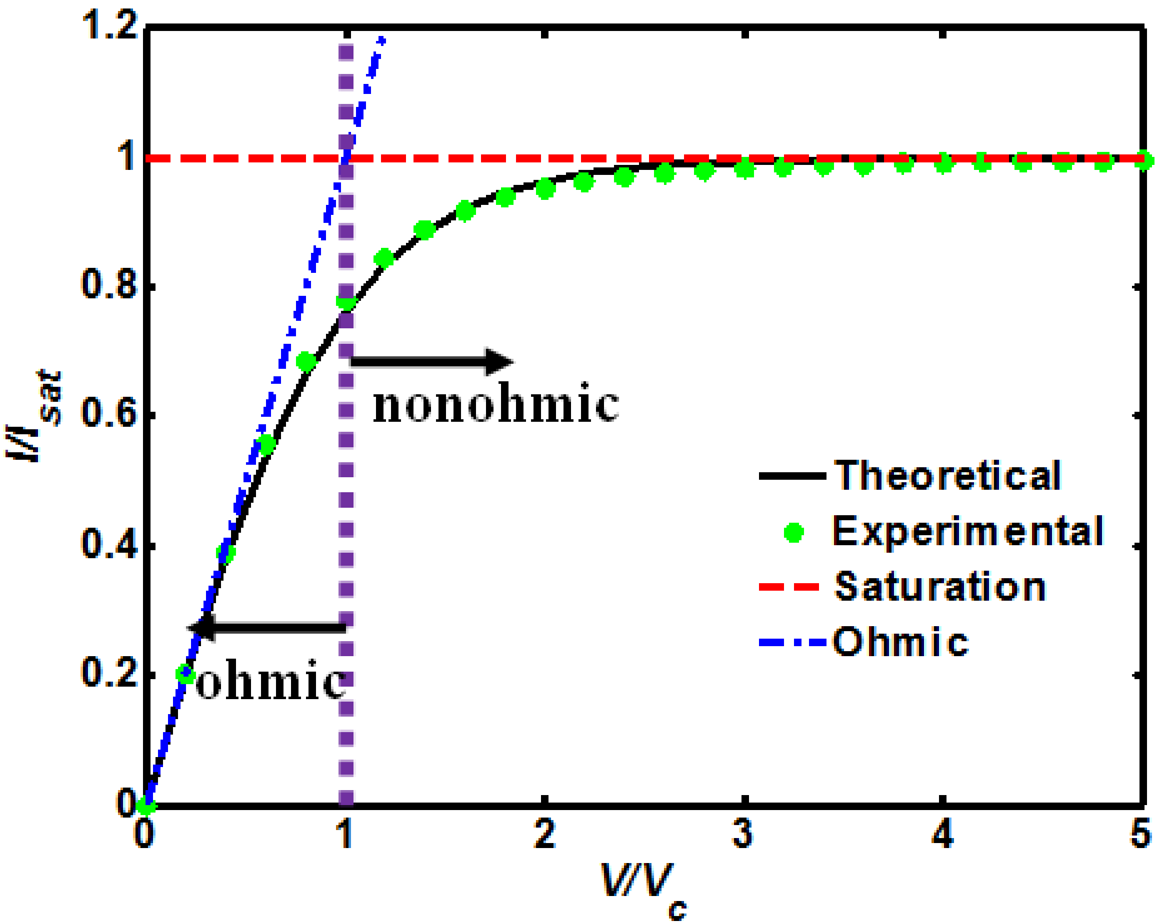
Sat Law (21st Century Paradigm)

Nonlinear Drift Response to the Electric Field

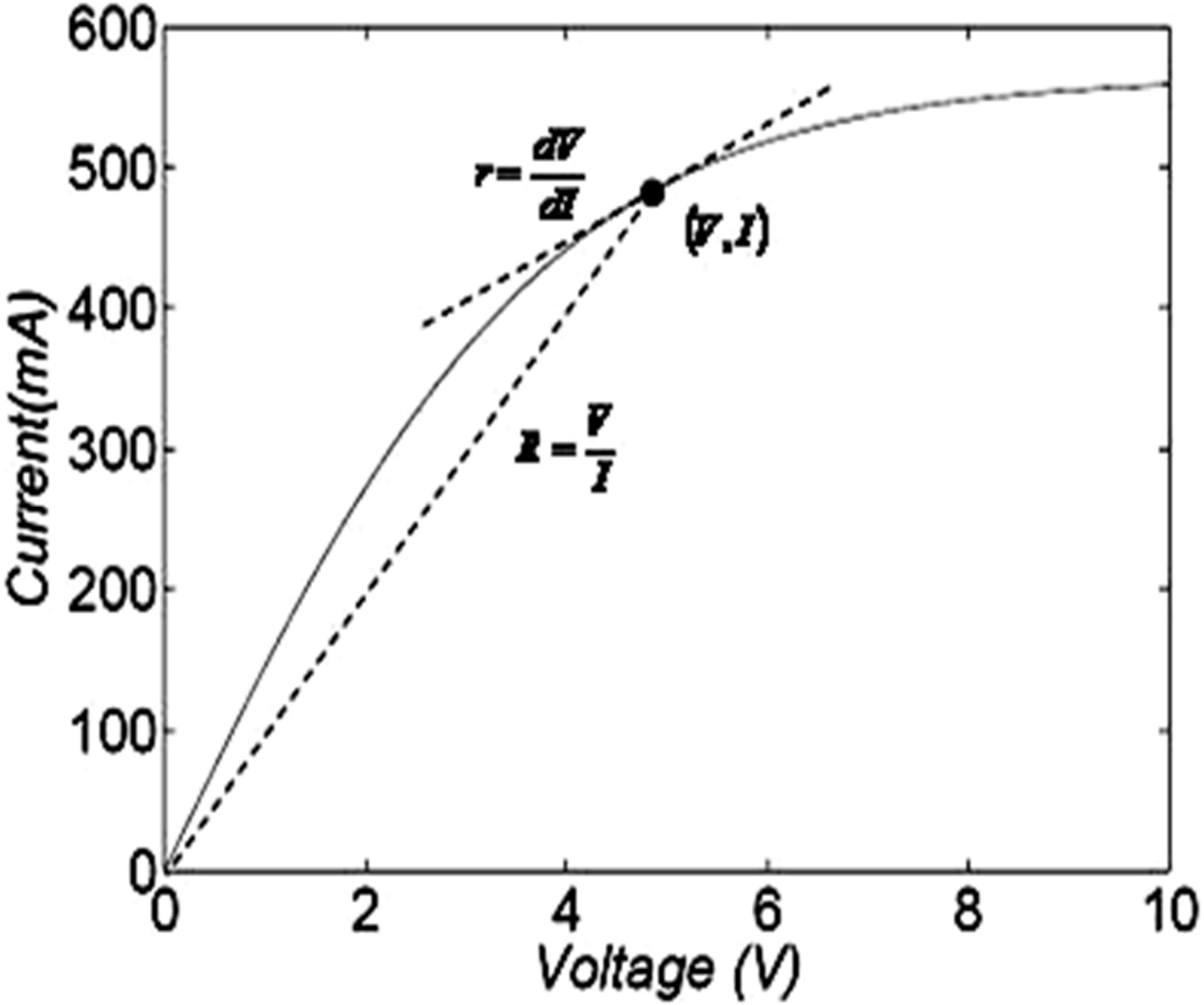
Empirical : $I_n = I_{sat} \dfrac{\dfrac{V}{V_c}}{\left[1+\left(\dfrac{V}{V_c}\right)^{\gamma}\right]^{\frac{1}{\gamma}}}$

$$I_n = \frac{V}{R_o} \frac{1}{\left[1+\left(\dfrac{V}{V_c}\right)^{\gamma}\right]^{\frac{1}{\gamma}}}$$

Current Response to Applied Potential



Direct and Incremental Resistance



Direct and Incremental Resistance Theory

$$\frac{R}{R_o} = \frac{V/V_c}{\tanh(V/V_c)} = \begin{cases} 1 & V < V_c \\ V/V_c & V >> V_c \end{cases}$$

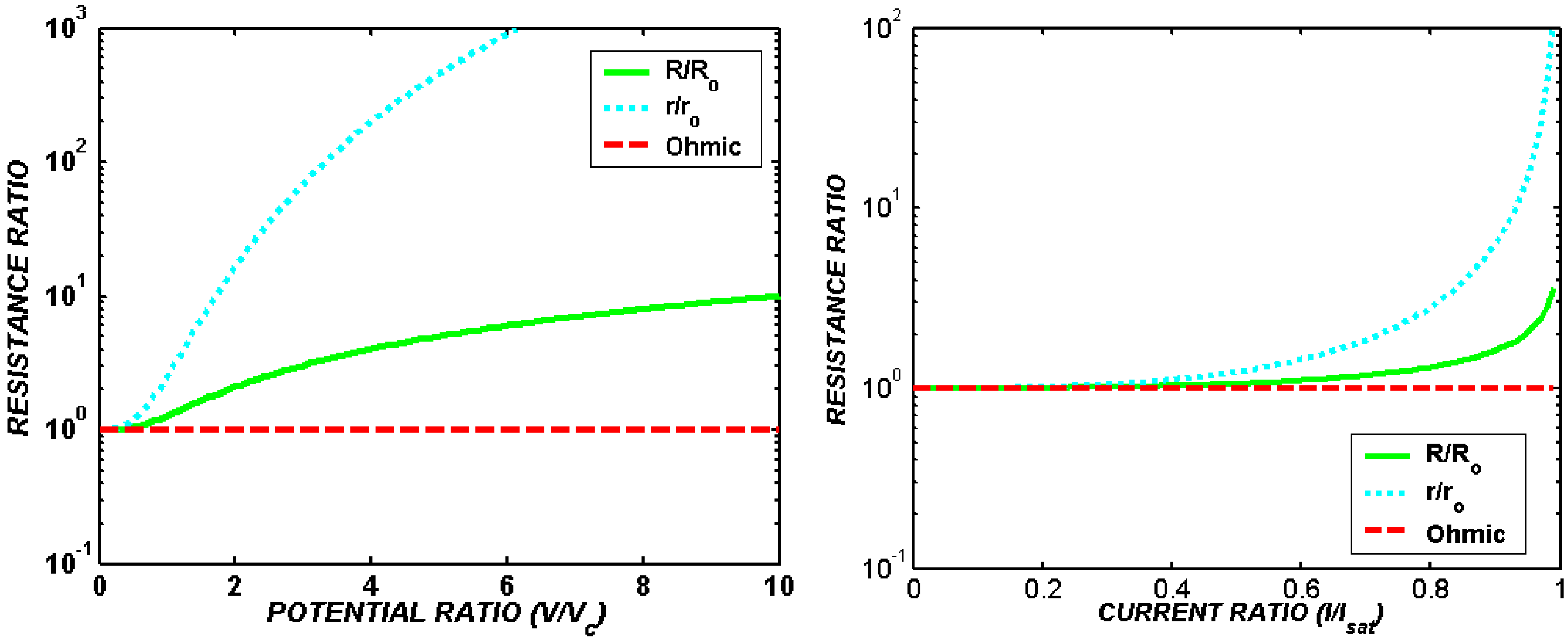
$$\frac{r}{R_o} = \cosh^2(V/V_c) = \begin{cases} 1 & V < V_c \\ \exp(2V/V_c)/4 & V >> V_c \end{cases}$$

Direct and Incremental Resistance Empirical

$$\frac{R}{R_o} = \left[1 + \left(\frac{V}{V_c}\right)^{\gamma}\right]^{\frac{1}{\gamma}} = \frac{1}{\left[1 - \left(\frac{I}{I_{sat}}\right)^{\gamma}\right]^{\frac{1}{\gamma}}}$$

$$\frac{r}{R_o} = \left[1 + \left(\frac{V}{V_c}\right)^{\gamma}\right]^{1+\frac{1}{\gamma}} = R_o \frac{1}{\left[1 - \left(\frac{I}{I_{sat}}\right)^{\gamma}\right]^{1+\frac{1}{\gamma}}} = \frac{R^{\gamma+1}}{R_o^{\gamma}}$$

Resistance Surge

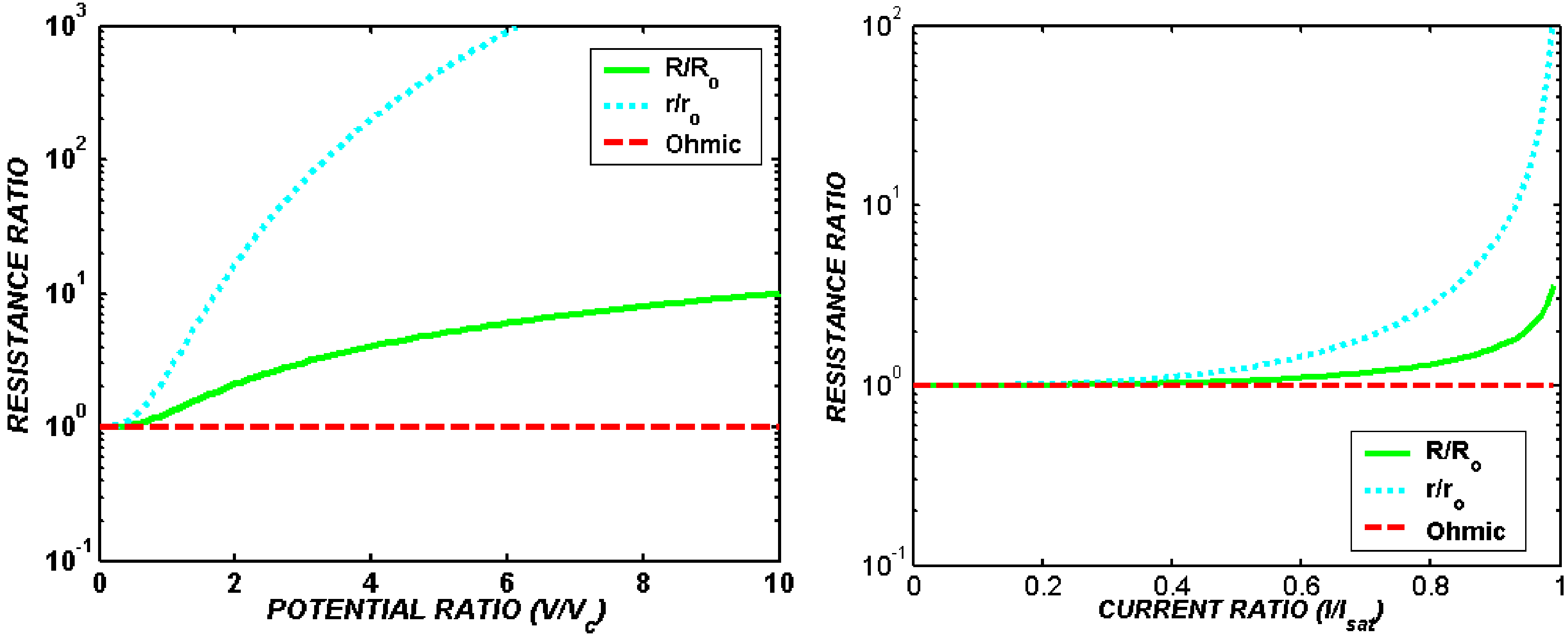


Modeling and Simulation of Microelectronic Devices

Topic 4: Charge Transport

Lecture 2

Resistance Surge



How Big is V_c?

$$V_c = \frac{v_{sat}}{\mu_{o\infty}} L = \frac{10^5\, m/s}{0.1 m^2 / V.s} L$$

$$V_c = 1.0 \frac{V}{\mu m} \begin{cases} 10\ kV & L = 1 cm \\ 1\ V & L = 1 \mu m \end{cases}$$

How Big is V_c in Ballistic Domain?

$$V_c = \frac{v_{sat}}{\mu_{oL}} L$$

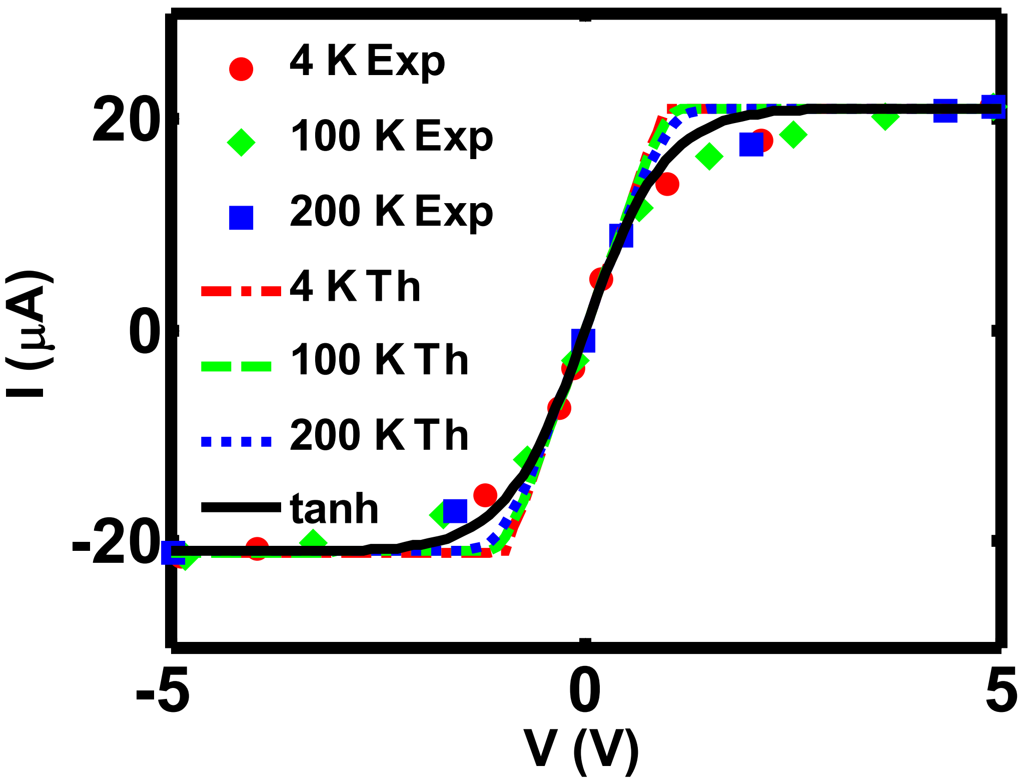
$$\mu_{oL} = \mu_{o\infty}\left(1 - e^{-\frac{L}{\ell_B}}\right) = \begin{cases} \mu_{o\infty} & L >> \ell_B \\ \mu_{o\infty}\frac{L}{\ell_B} & L << \ell_B \end{cases}$$

$$V_c = \frac{v_{sat}}{\mu_{o\infty}\left(1 - e^{-\frac{L}{\ell_B}}\right)} L = \begin{cases} \frac{v_{sat}}{\mu_{o\infty}} L & L >> \ell_B \\ \frac{v_{sat}}{\mu_{o\infty}} \ell_B & L << \ell_B \end{cases}$$

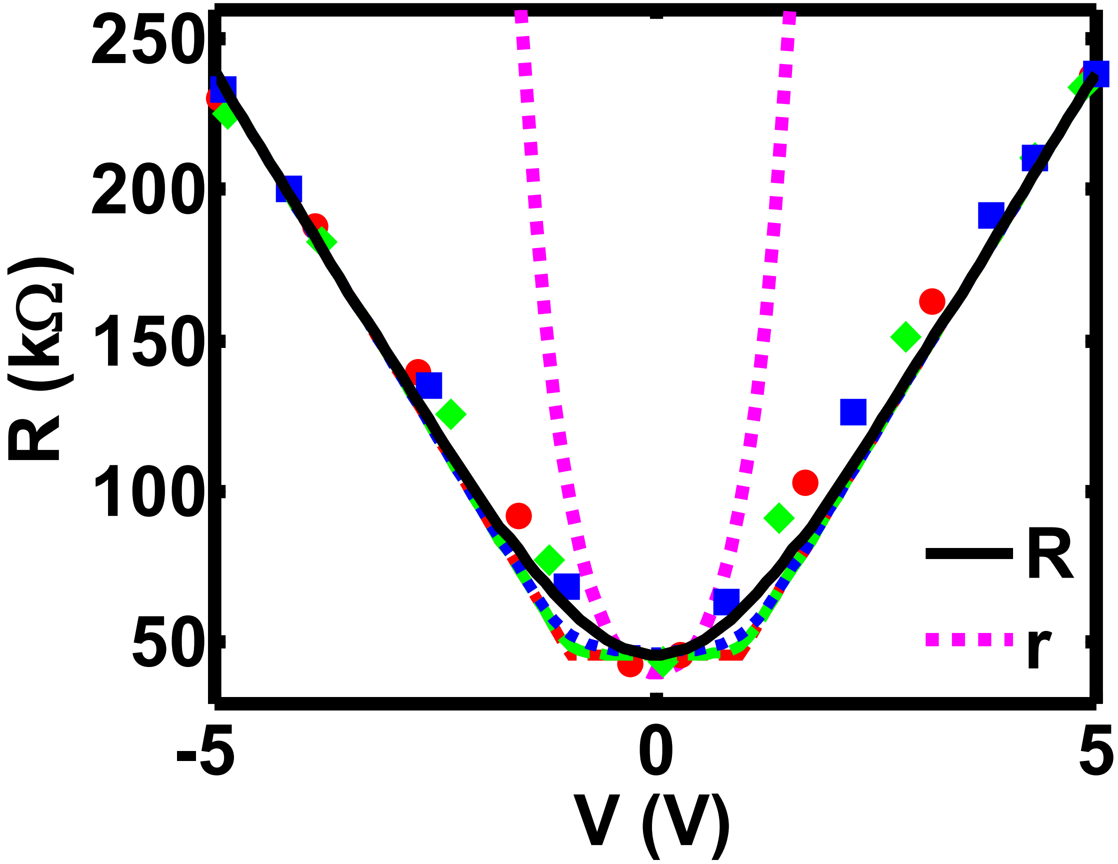
Critical voltage controlled by ballistic mfp!!!

Metallic CNT-Application

$I_{sat} = 21\ \mu A \qquad R_o = 40\ k\Omega$



Metallic CNT-Resistance Surge



Power Law

$$P = VI = \frac{VV_c}{R_o}\tanh\left(\frac{V}{V_c}\right) = VI_{sat}\tanh\left(\frac{V}{V_c}\right)$$

$$P = \frac{V^2}{R_o} \quad V < V_c (Ohmic)$$

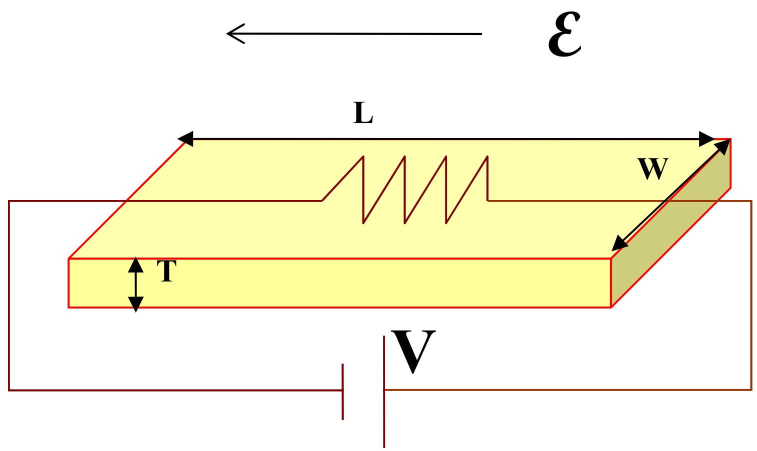
$$P = \frac{VV_c}{R_o} \quad V >> V_c$$

2D and 1D Resistors

2D Channel

$$I = \frac{Q}{t} = \frac{n_2 (LW)(q)}{t}$$

$$I = n_2 \frac{L}{t} qW = n_2 v_D qW$$



1D Channel

$$I = \frac{Q}{t} = \frac{n_1 Lq}{t}$$

$$I = n_1 \frac{L}{t} q = n_1 v_D q$$

Ohm's Law (20th Century Paradigm)
2D Resistor

$$R_o = \rho_2 \frac{L}{W}$$

$$\rho_2 = \frac{1}{\sigma_2} = \frac{1}{\left(n_2\mu_{on} + p_2\mu_{op}\right)q}$$

Sheet Resistivity

$\rho_2 = \rho_3 / T \approx \rho_3 / x_j$ $(unit : \Omega / \square)$

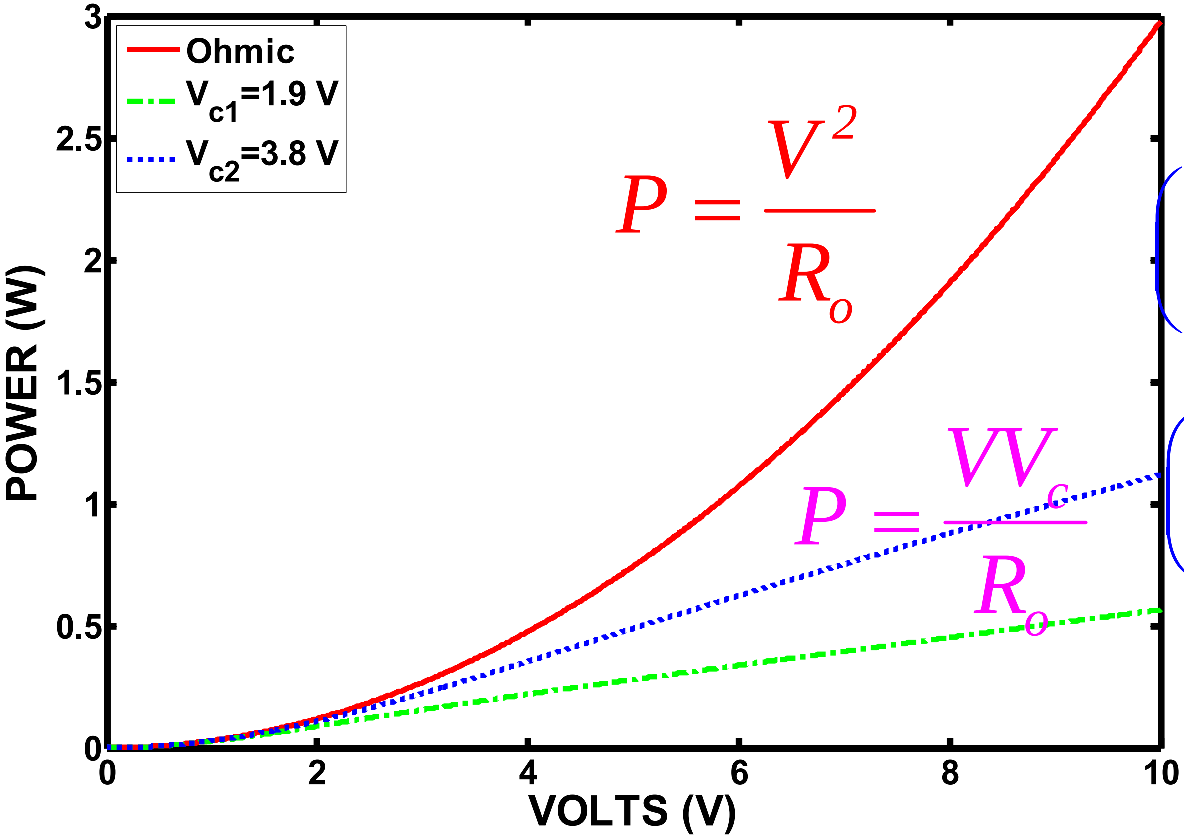
$$R_o = \rho_2 \text{ if } W = L$$

Ohm's Law (20th Century Paradigm) 1D Resistor

$$R_o = \rho_1 L$$

$$\rho_1 = \frac{1}{\sigma_1} = \frac{1}{\left(n_1 \mu_{on} + p_1 \mu_{op}\right) q} \quad (unit : \Omega / m)$$

Power Law Display



$$R_o = \frac{\rho_2}{(W/L)} = 33.6\ \Omega$$

$$\left(\frac{W}{L}\right)_1 = \frac{100\ \mu m}{5\ \mu m} = 20$$

$$\left(\frac{W}{L}\right)_2 = \frac{200\ \mu m}{10\ \mu m} = 20$$

$$\mathcal{E}_c = 3.8\ \frac{kV}{cm}$$

$$V_{c1} = 1.9\ V$$

$$V_{c2} = 3.8\ V$$

How Big is V_c?

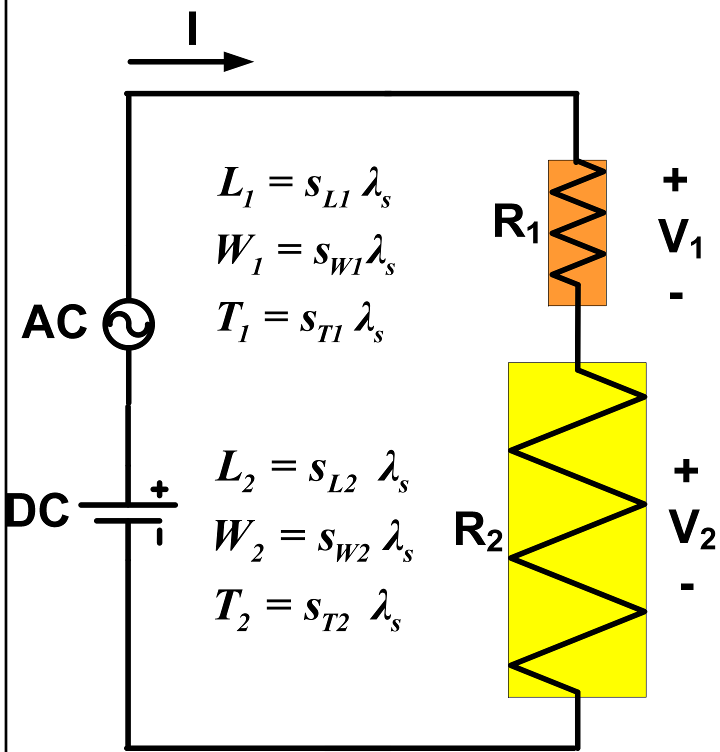
$$V_c = \frac{v_{sat}}{\mu_{o\infty}} L = \frac{10^5\, m/s}{0.1 m^2 / V.s} L$$

$$V_c = 1.0 \frac{V}{\mu m} \begin{cases} 10\ kV & L = 1 cm \\ 1\ V & L = 1 \mu m \end{cases}$$

Voltage Division

$$V_{c1}\tanh\left(\frac{V_1}{V_{c1}}\right)=V_{c2}\tanh\left(\frac{V-V_1}{V_{c2}}\right)$$

$$R_{o1} = R_{o2} = 67.2\ \Omega$$



$L_1 = s_{L1}\,\lambda_s$

$W_1 = s_{W1}\lambda_s$

$T_1 = s_{T1}\,\lambda_s$

$L_2 = s_{L2}\,\lambda_s$

$W_2 = s_{W2}\,\lambda_s$

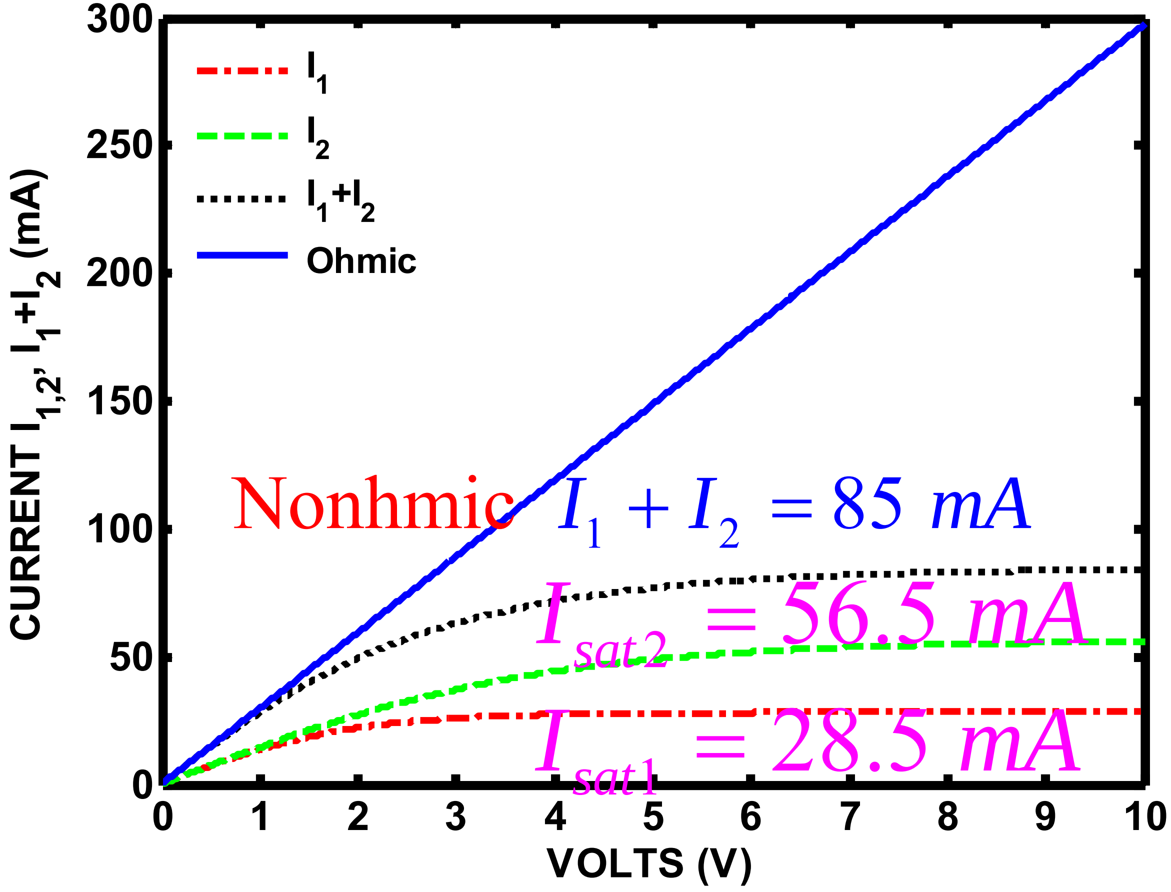
$T_2 = s_{T2}\,\lambda_s$

$$\left(\frac{W}{L}\right)_1 = \frac{50\ \mu m}{5\ \mu m} = 10$$

$$\left(\frac{W}{L}\right)_2 = \frac{100\ \mu m}{10\ \mu m} = 10$$

Current Division

Ohmic $I_1 + I_2 = 300\ mA$

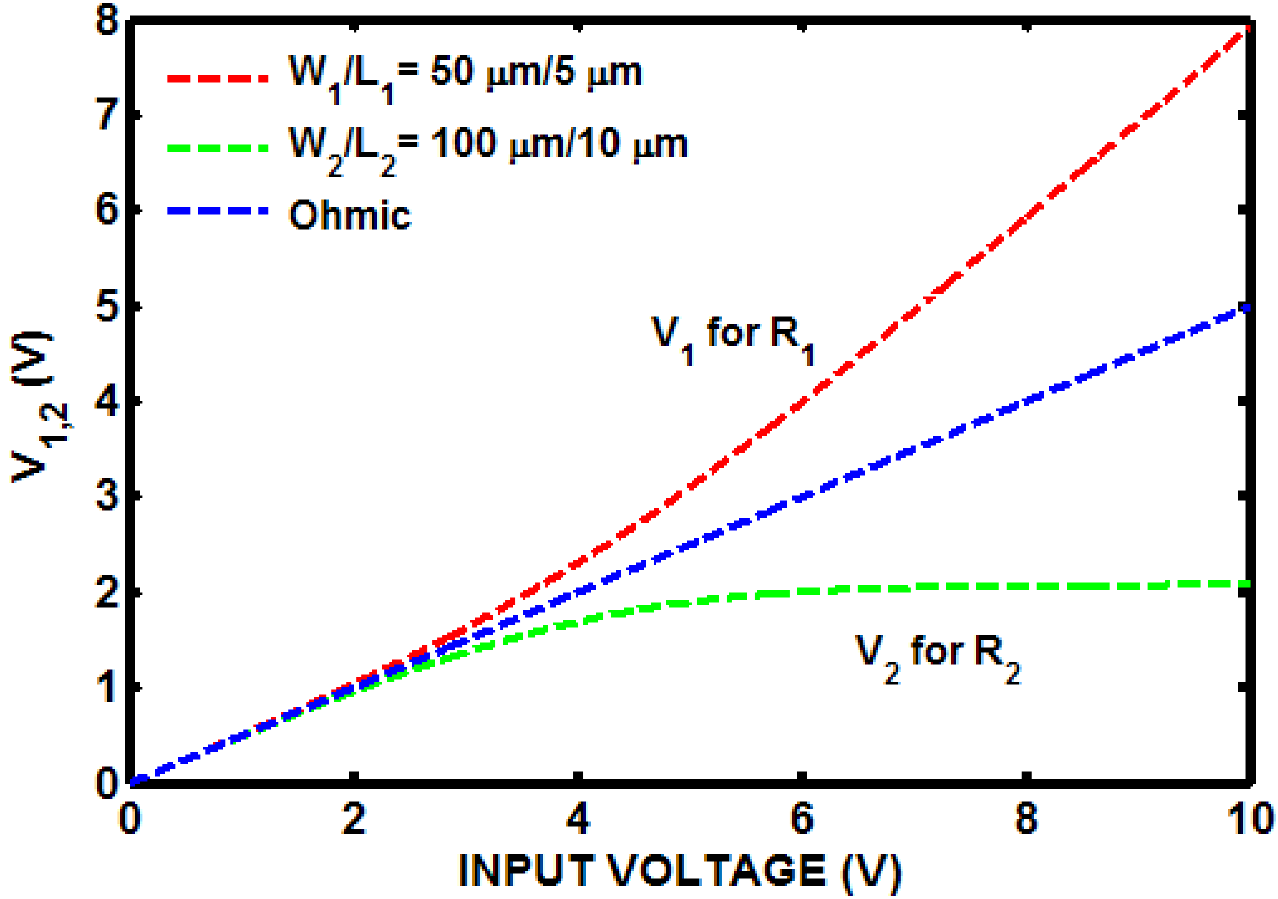


Modeling and Simulation of Microelectronic Devices

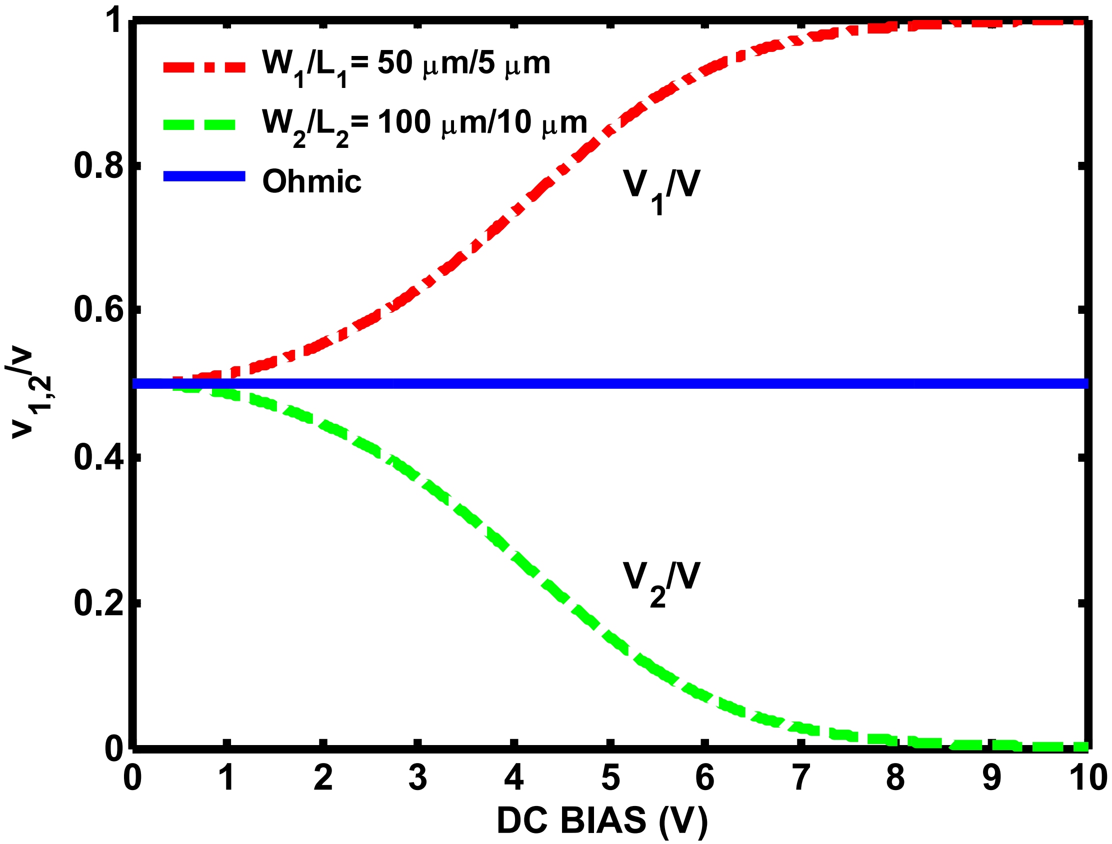
Topic 4: Charge Transport

Lecture 3

Voltage Division



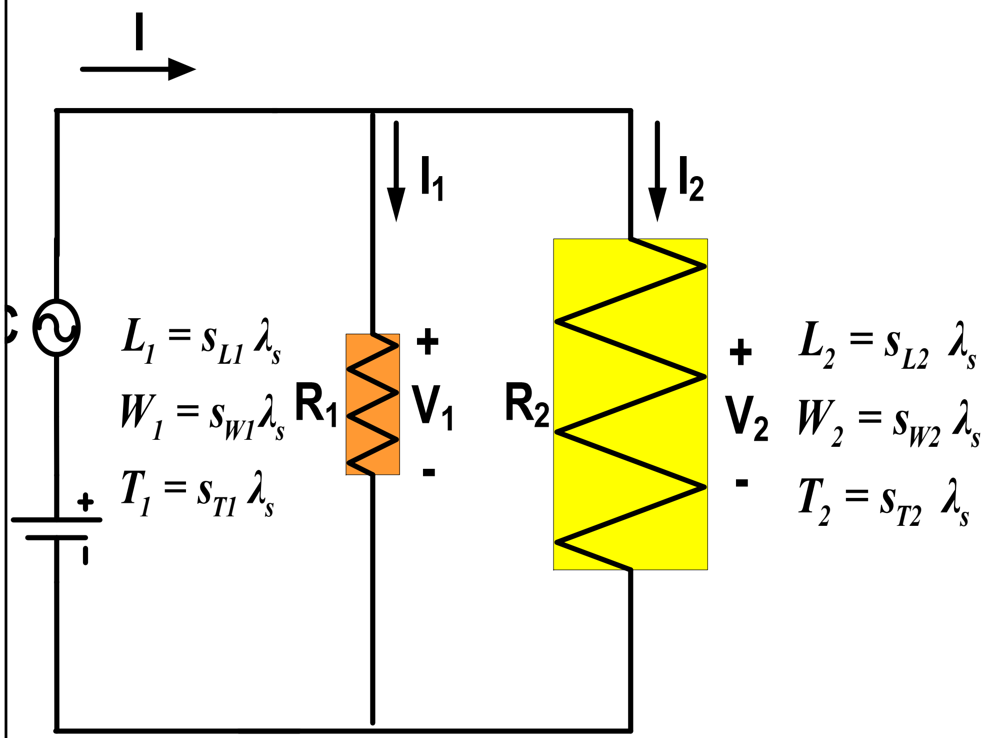
Signal Voltage Division



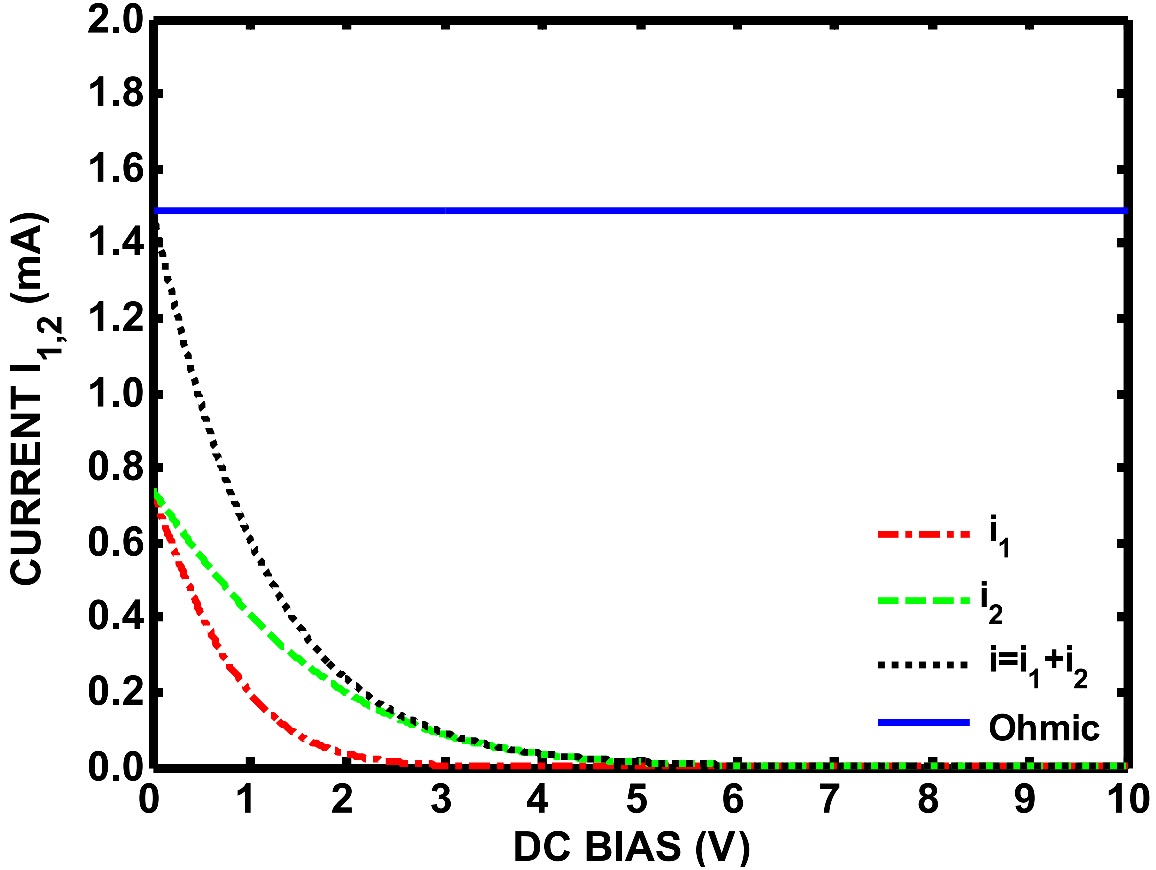
Current Division

$$R_{o1} = R_{o2} = 67.2\ \Omega$$

$$\left(\frac{W}{L}\right)_1 = \frac{50\ \mu m}{5\ \mu m} = 10 \qquad \left(\frac{W}{L}\right)_2 = \frac{100\ \mu m}{10\ \mu m} = 10$$



Signal Current Division

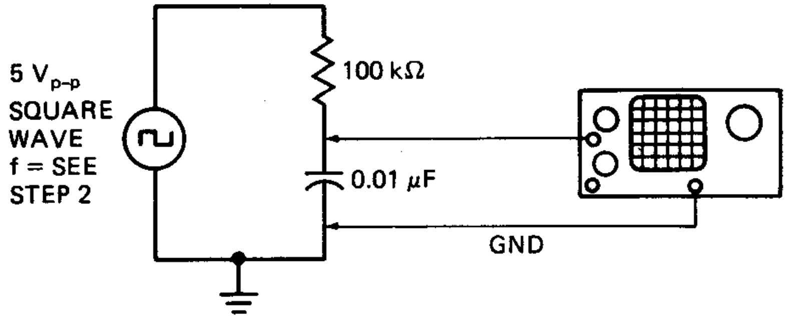


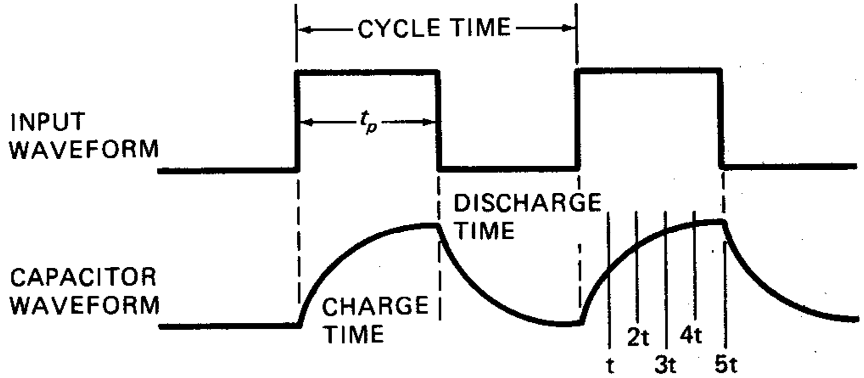
Speed Inhibiting factors

Transit Time (*Gate*) Delay: $\tau_t = \frac{L}{v}$

RC Time (*Wire*) Delay: $\tau_{RC} = RC$

RC-Time Delay-Ohmic





$$V = iR_o + \frac{q}{C} \Rightarrow \frac{dq}{dt} + \frac{q}{R_o C} = \frac{V}{R_o} \quad I.C.:\ q(0) = 0$$

$$q(t) = CV\left(1 - e^{-\frac{t}{\tau_o}}\right) \Rightarrow v_c(t) = V\left(1 - e^{-\frac{t}{\tau}}\right) \quad \tau_o = R_o C$$

$$C = \varepsilon_o \varepsilon_r \frac{A}{d}$$

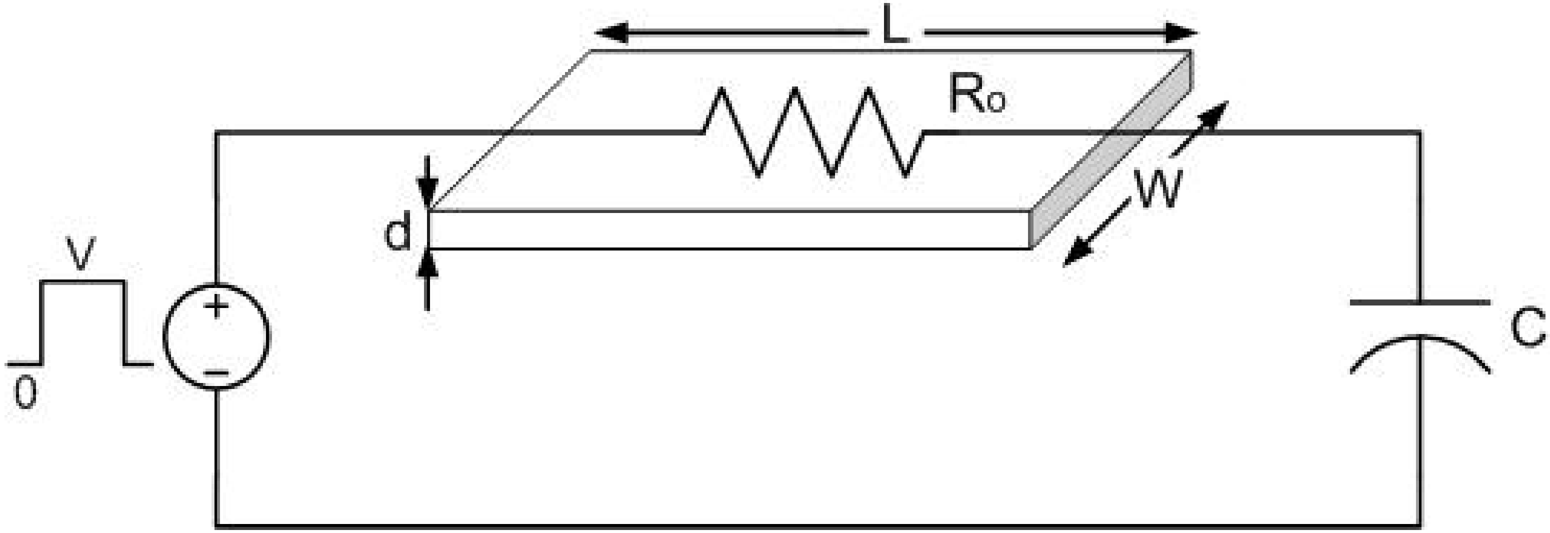
$$R_o = \rho \frac{L}{A} \ (3D)$$

$$= \rho_s \frac{L}{W} (Q2D)$$

$$= \rho_\ell L \quad (Q1D)$$

Low resistivity (ρ) and low-ε_r (low-K) are desirable

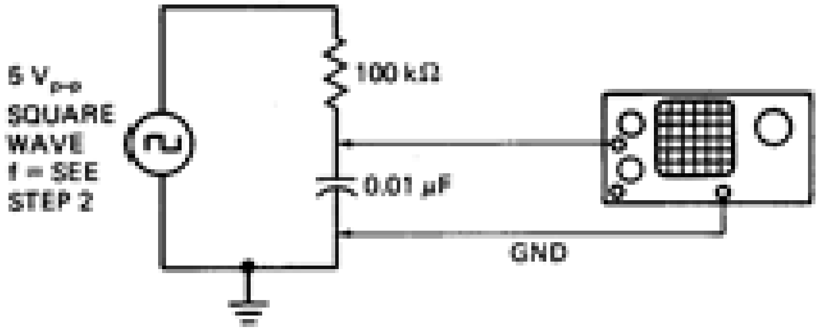
RC Time Delay

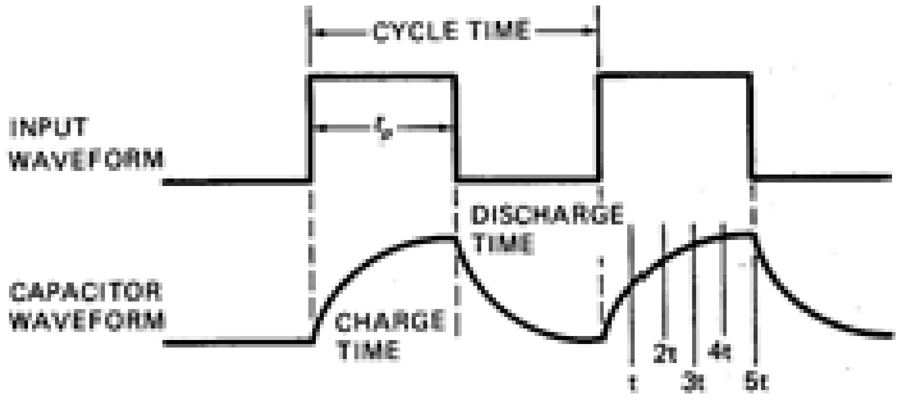


$$v_c(t) = V(1 - e^{-t/R_oC})$$

$$v_c(t = R_oC) = V(1 - e^{-1}) = 0.63\ V$$

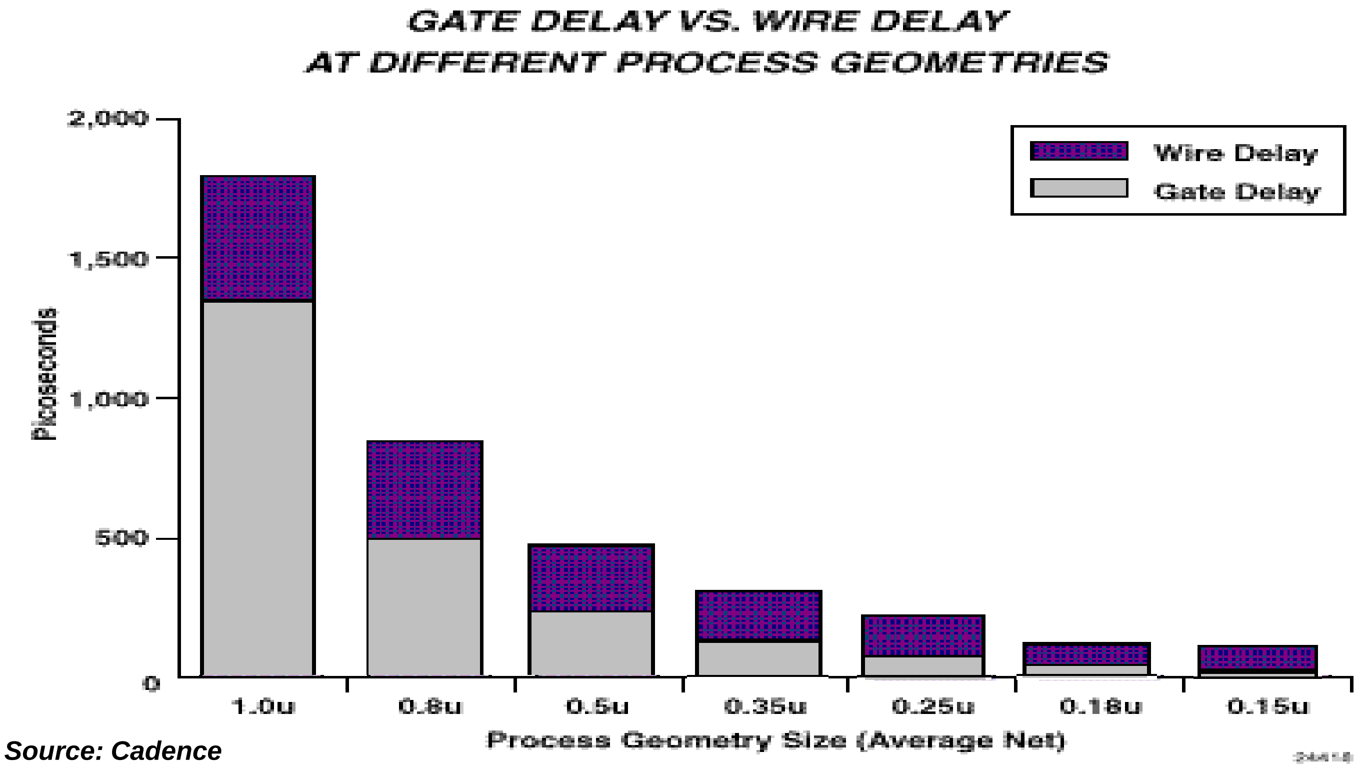
Switching Delay





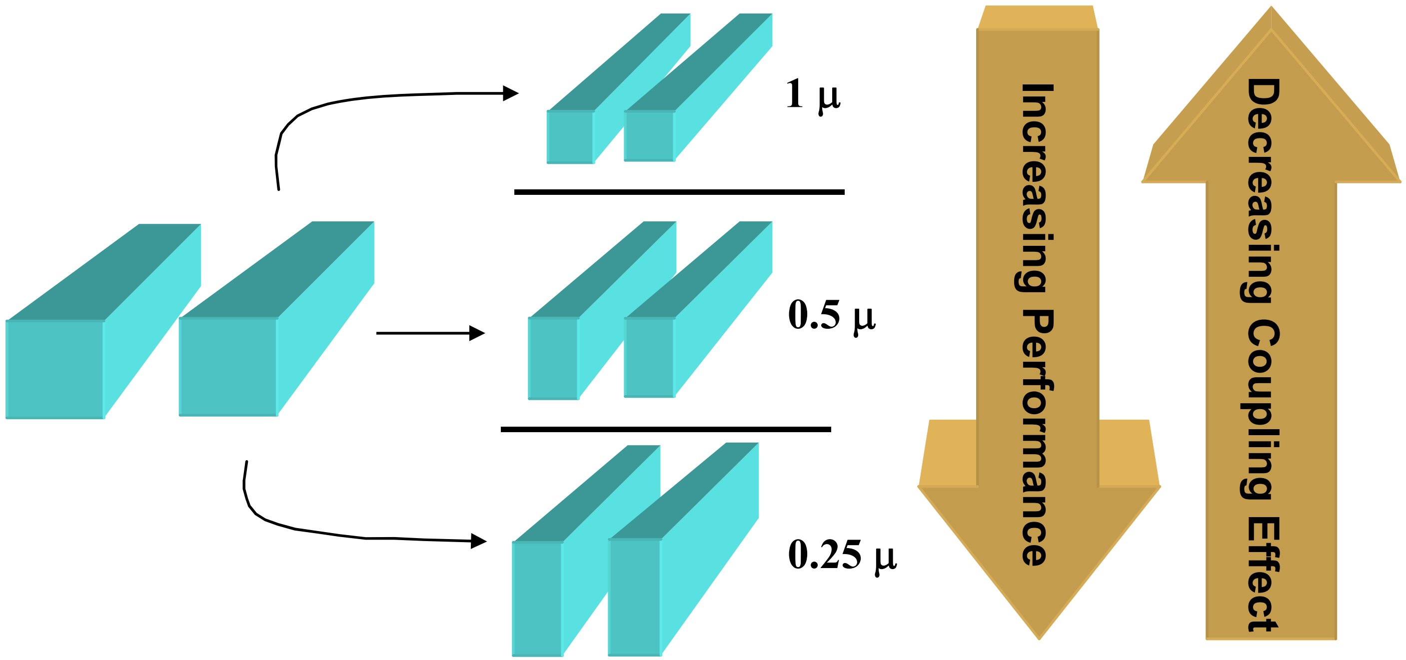
Switching Time: $t_{10\%} - t_{90\%} = 2.2 \; RC$

RC and Transit Time Delays



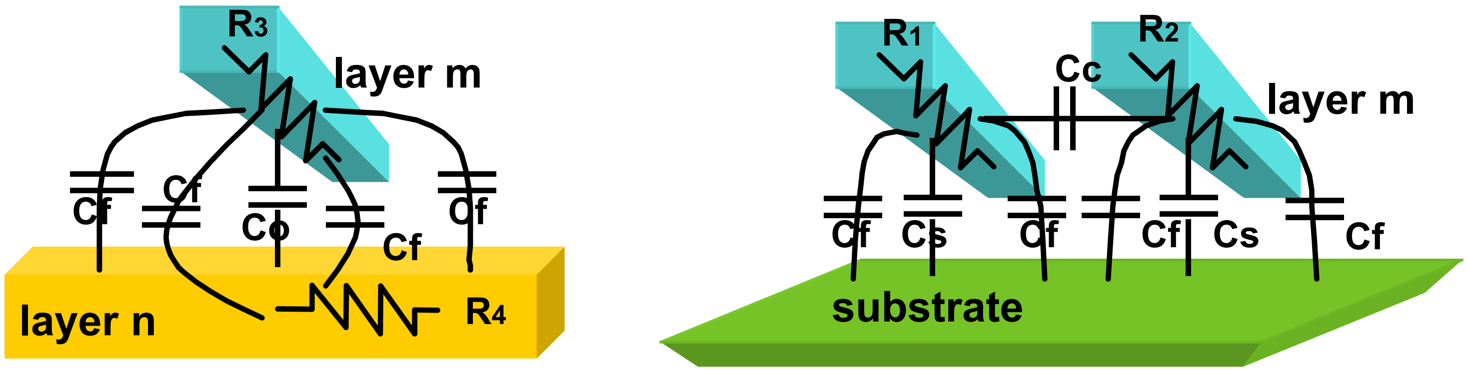
Source: Cadence

Interconnect Performance



Increased cross-section improves performance but also increases noise and capacitive and inductive coupling

RC Delay Considerations



Cint = Cf + Cs + Co + Cload

τ = Rint * (Cint + Cc/(Cint+ Cc))

τ = Rint * (Cint^2 + Cint.Cc +Cc)/(Cint + Cc)

- Cc depends on dimensional shrink due to increased in cross-section
- In VLSI, make Cc becomes insignificant as possible, then

τ = Rint * Cint

Goals for High Speed Performance

- Large transistor current
  - Time constants
  - Interconnects
  - Cross talk

- Reduced transit time
  - Increased Mobility
  - High Saturation Velocity
  - Reduced Size

Transit Time Delay

$$\frac{\tau_t}{\tau_{to}} = \frac{V / V_c}{\tanh(V / V_c)}$$

$$\tau_t = \frac{L}{v_{sat} \tanh(V / V_c)} \approx \begin{cases} \tau_{to} = L^2 / \mu_o V & V < V_c \\ \tau_{t\infty} = L / v_{sat} & V >> V_c \end{cases}$$

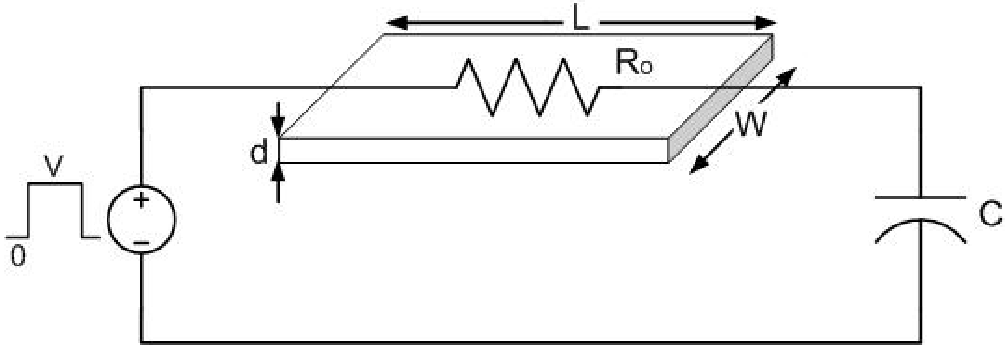
Transit-Time Delay

$$\tau_t = \frac{L}{v_d} = \frac{L}{\mu_o \mathcal{E}} = \frac{L^2}{\mu_o V} = \frac{(1\,\mu m)^2}{0.8\,\frac{m^2}{V.s}\,5V} = 0.25\,ps\ (Ohmic)$$

$$\tau_t = \frac{L}{v_{sat}} = \frac{1\ \mu m}{10^5\,m/s} = 10\,ps\ (Saturation)$$

$$\tau = R_o C = 1\Omega \times 0.01\ pF = 0.01ps << t_{ttd}$$

RC-Time Delay-Nonohmic



$$I = I_{sat}\tanh\left(\frac{V}{V_c}\right) = \frac{V_c}{R_o}\tanh\left(\frac{V}{V_c}\right) = \frac{V}{R_o}\frac{\tanh\left(\frac{V}{V_c}\right)}{\frac{V}{V_c}}$$

$$R_o = \rho\frac{L}{A}\ (3D)$$

$$= \rho_s\frac{L}{W}\ (Q2D)$$

$$= \rho_\ell L\ \ (Q1D)$$

$$I_{sat} = nqv_{sat}A = n_s qv_{sat}W = \frac{V_c}{R_o}$$

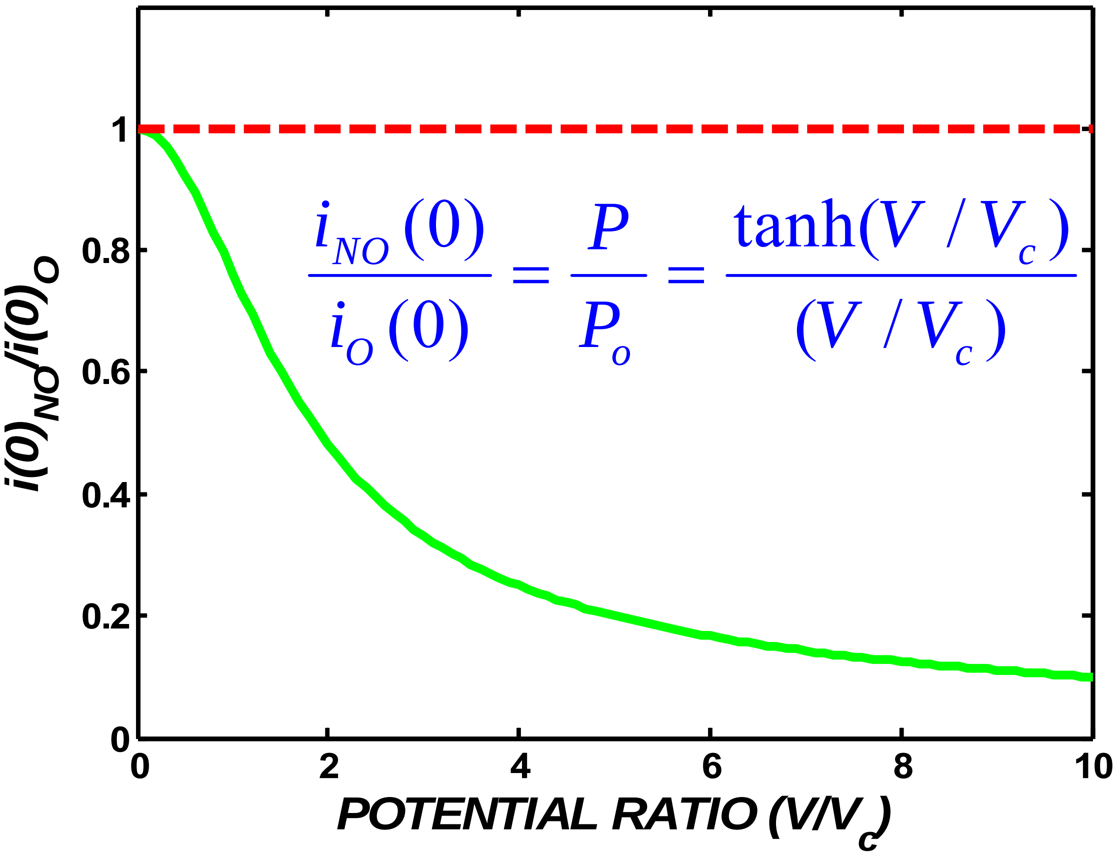
Transients for TTD

$$i_R(t) = I_{sat} \tanh(\frac{v_R(t)}{V_c})$$

$$V = v_R(t) + v_C(t) = V_c \tanh^{-1}\left(\frac{i_R(t)}{I_{sat}}\right) + \frac{q(t)}{C}$$

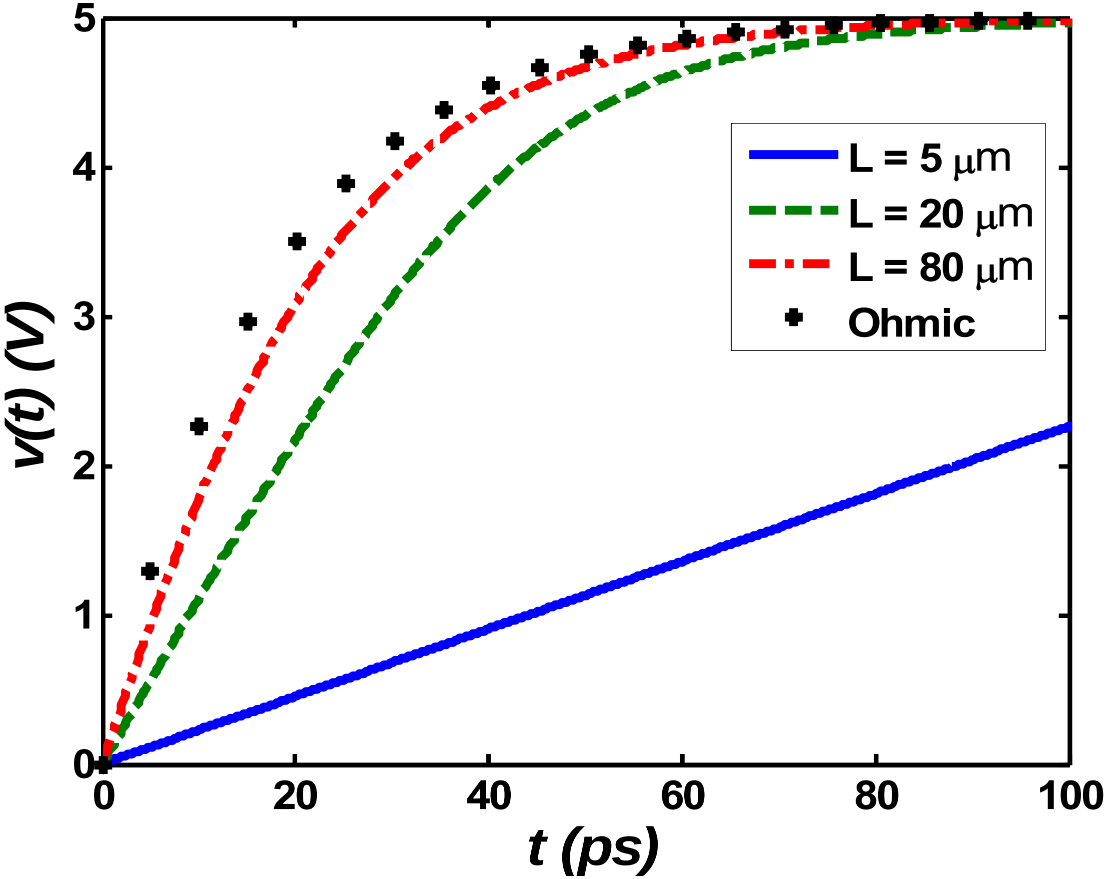
Initial Condition $i_R(0) = I_{sat} \tanh\left(\frac{V}{V_c}\right)$

Nonohmic to Ohmic Ratio



$$\frac{i_{NO}(0)}{i_{O}(0)} = \frac{P}{P_o} = \frac{\tanh(V/V_c)}{(V/V_c)}$$

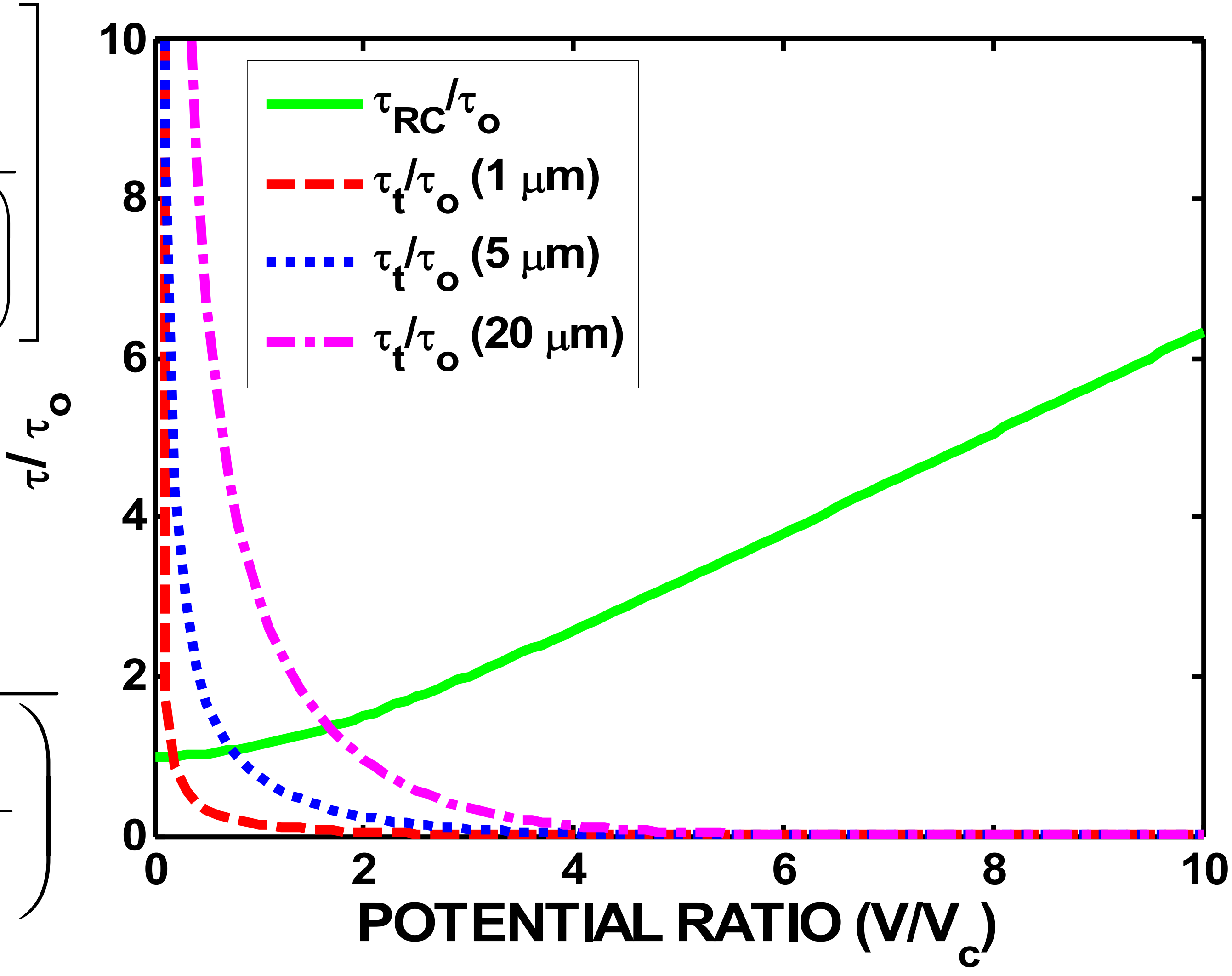
Voltage Transients



RC Delay Enhancement

$$\frac{\tau_{RC}}{\tau_o} = \ln\left[\frac{\sinh\left(\frac{V}{V_c}\right)}{\sinh\left(\frac{V}{eV_c}\right)}\right]$$

$$R = R_o \frac{\frac{V}{V_c}}{\tanh\left(\frac{V}{V_c}\right)}$$



L/R Time Constant

$$V = v_R(t) + v_L(t) = V_c \tanh^{-1}\left(\frac{i(t)}{I_{sat}}\right) + \mathscr{L}\frac{di}{dt}$$

$$\tau_{L/R_o}\frac{di}{dt} + I_{sat}\tanh^{-1}\left(\frac{i}{I_{sat}}\right) = \frac{V}{R_o}$$

$$i(0) = 0 \quad , \quad i(\infty) = I_{sat}\tanh\left(\frac{V}{V_{cr}}\right)$$

Nonohmic Versus Ohmic L/R

$$\frac{\tau_{L/R}}{\tau_{L/R_o}} = \frac{1+(1-e^{-1})(V/V_c)}{\left(1+V/V_c\right)^2}$$