UTM
UNIVERSITI TEKNOLOGI MALAYSIA

SKEE1223: Digital Electronics

4 – Logic Gates and Boolean Algebra

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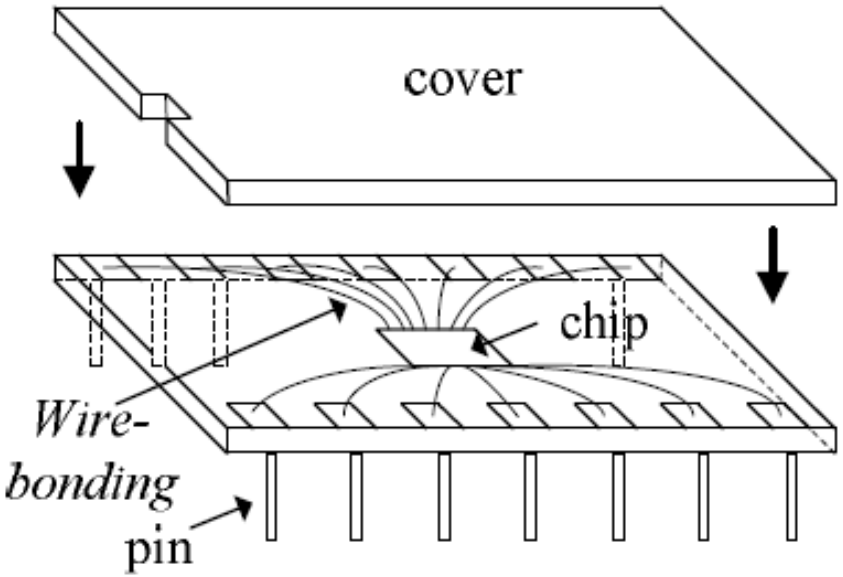
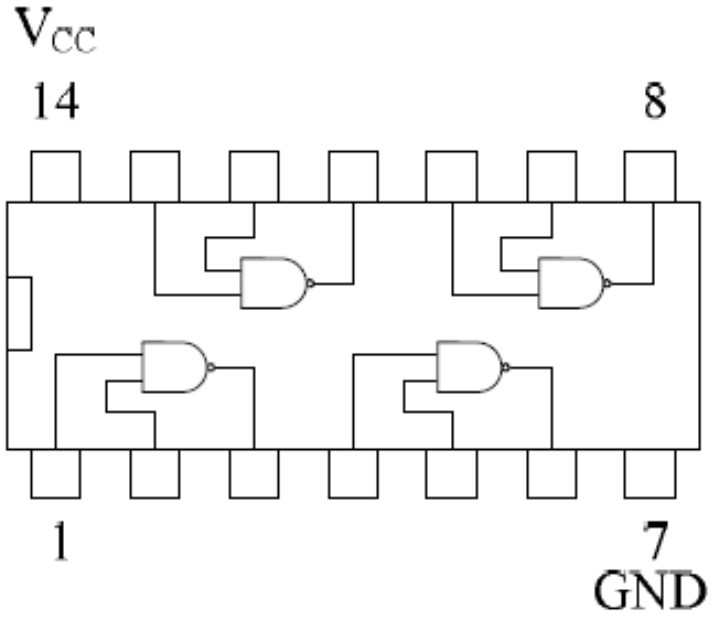
Gates packaging

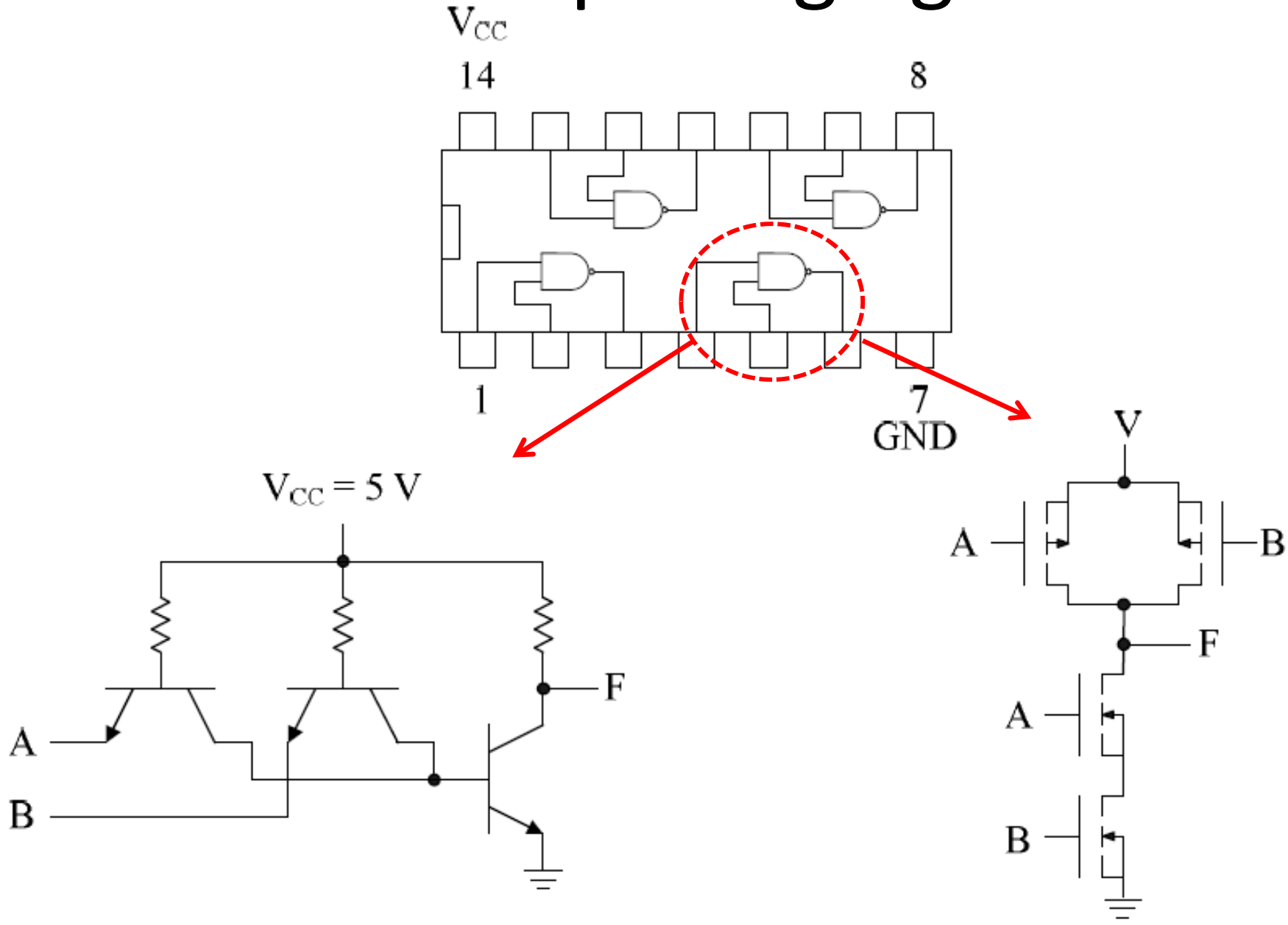


(a) packaging



(b) pin diagram 7400

Gates packaging



Exercise 1: Least Challenging

Complete the sentences:

- If all inputs of an OR gate are low, the output is_____
- If any input of an AND gate is low, the output is _____
- The output of a NOT gate is always the _____ of the input.

Logic Gates and Boolean Algebra

- Logic Gates
  - Inverter, OR, AND, Buffer, NOR, NAND, XOR, XNOR
- Boolean Theorem
  - Commutative, Associative, Distributive Laws
  - Basic Rules
- DeMorgan's Theorem
- Universal Gates
  - NAND and NOR
- Canonical/Standard Forms of Logic
  - Sum of Product (SOP)
  - Product of Sum (POS)
  - Minterm and Maxterm

Boolean Algebra

- Mathematics of digital system
- Important in the analysis of logic circuit
- Binary variable– a symbol (letter) used to represent a logical quantity.
- Example binary variables and their values:
  - A = 0
  - B = 1
- Complement—the inverse of a variable
  - The complement of A is $\overline{A}$ or A'.

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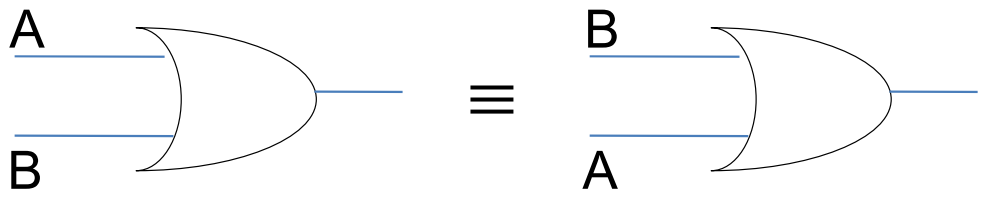
Commutative Law

- Order independent
- For addition : $X+Y = Y+ X$
- For multiplication: $XY = YX$

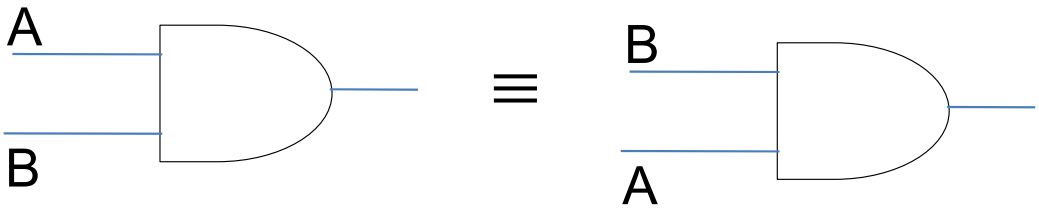
Commutative Law

In terms of the result, the order in which variables are ORed or ANDed makes no difference.

$A + B = B + A$



$AB = BA$



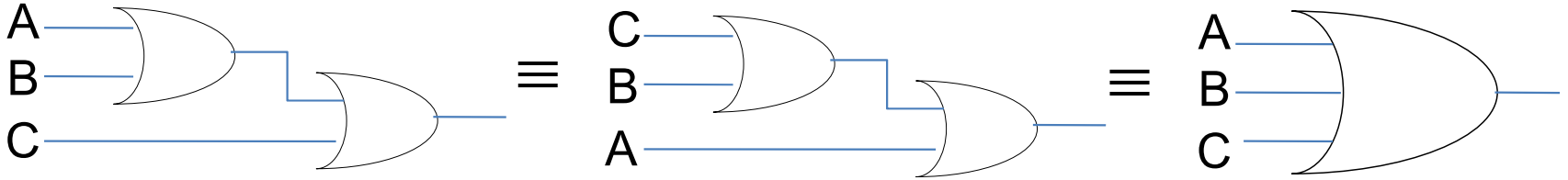
Associative Law

- Grouping is order-independent
- For addition : $X+(Y+Z) = (X+Y)+Z$
- For multiplication: $X(YZ) = (XY)Z$

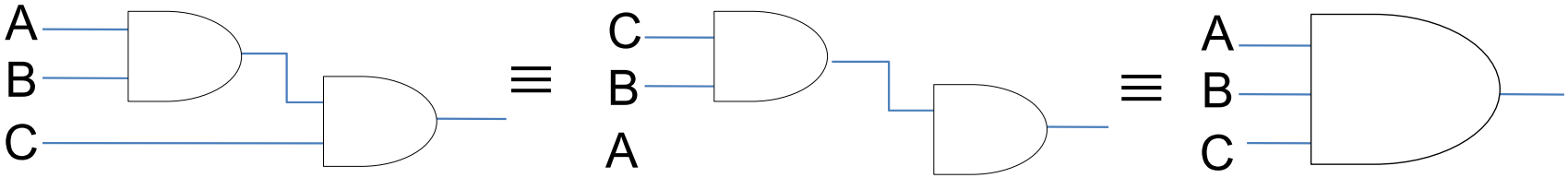
Associative Law

When ORing or ANDing more than two variables, the result is the same regardless of the grouping of the variables.

$$A + (B + C) = (A + B) + C$$



$$A(BC) = (AB)C$$

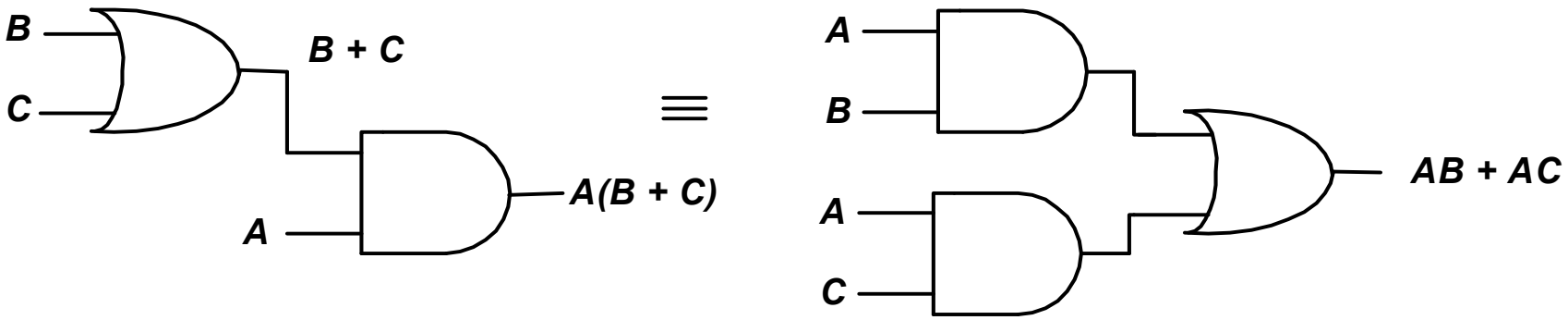


Distributive Law

Expanding an expression by multiplying term by term

X(Y+Z) = XY + XZ

$$A(B + C) = AB + AC$$



Distributive Law

A common variable can be factored from an expression just as in ordinary algebra.

$$AB + AC = A(B + C)$$

12 Boolean Rules

1. $A + 0 =$
2. $A + 1 =$
3. $A * 0 =$
4. $A * 1 =$
5. $A + A =$
6. $A + \overline{A} =$
7. $A \bullet A =$
8. $A \bullet \overline{A} =$
9. $\overline{\overline{A}} =$
10. $A+AB =$
11. $A+\overline{A}B =$
12. $(A+B)(A+C) =$

12 Boolean Rules

1. $A + 0 = A$
2. $A + 1 = 1$
3. $A * 0 = 0$
4. $A * 1 = A$
5. $A + A = A$
6. $A + \overline{A} = 1$
7. $A \bullet A = A$
8. $A \bullet \overline{A} = 0$
9. $\overline{\overline{A}} = A$
10. $A+AB = A$
11. $A+\overline{A}B = A+B$
12. $(A+B)(A+C) = A + BC$

Boolean Simplification - Example

- Using boolean theorem, Simplify the expression:

$$AB + A(B + C) + B(B + C)$$

Apply distributive law,

$$AB + AB + AC + BB + BC$$

Apply rule 7 (BB = B), and rule 5 (AB + AB = AB)

$$AB + AC + B + BC$$

Apply rule 10 (B + BC = B)

$$AB + AC + B$$

Boolean Simplification - Example

$$AB + AC + B$$

Apply rule 10 (AB + B = B)

$$B + AC$$

At this point, the expression is simplified as much as possible

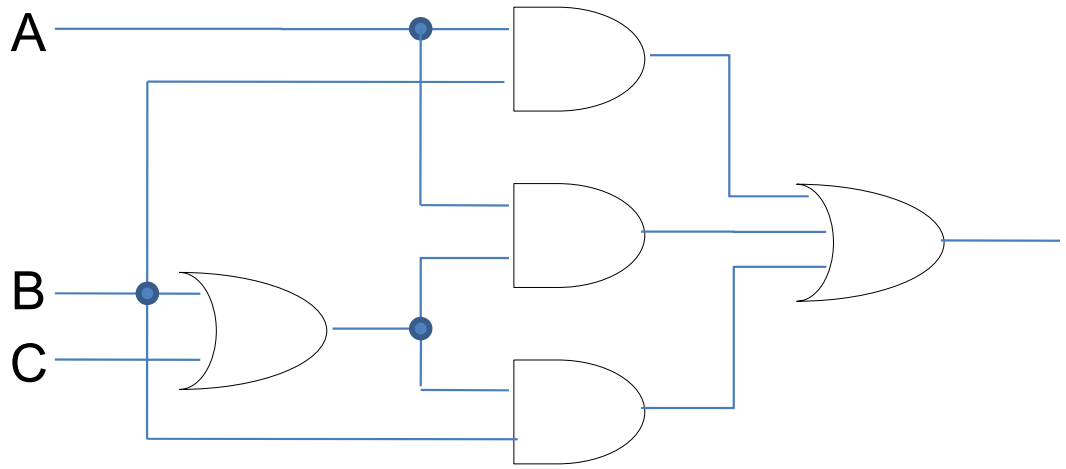
Original expression is $AB + A(B + C) + B(B + C)$

Which is logically equal to $B + AC$

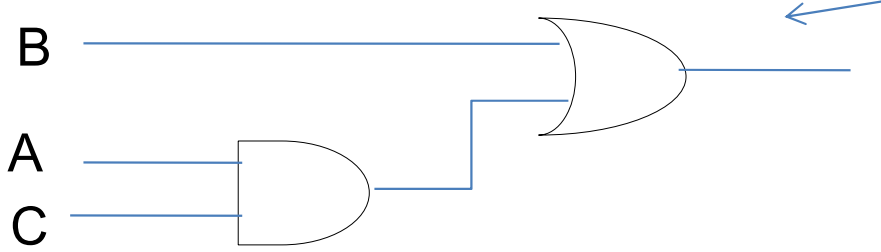
In terms of design, what is the advantage of Boolean simplification?

Boolean Simplification - Example

Original expression is $AB + A(B + C) + B(B + C)$



Which is logically equal to $B + AC$



Faster
Compact design
Lower cost

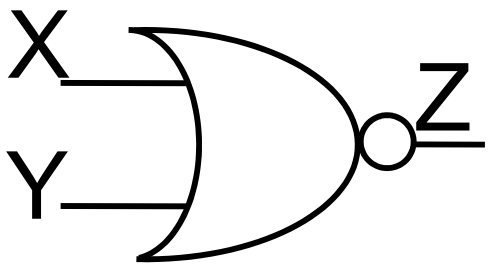
Boolean Simplification - Example

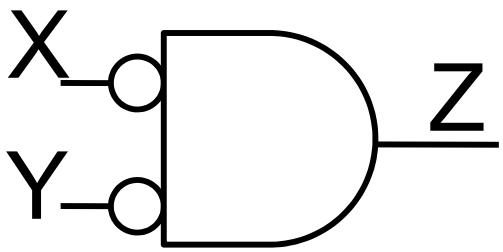
- Applying boolean theorem for logic simplification depends on a thorough knowledge of boolean algebra, with some ingenuity and cleverness
- Please look at Floyd's book examples 4-10, 4-11, and 4-12, as well as some exercises in the book to gain experience

DeMorgan's Theorem

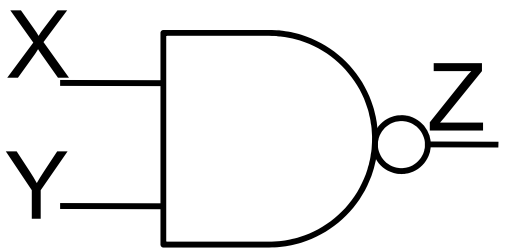
- DeMorgan proposed two theorems that are an important part of Boolean Algebra.

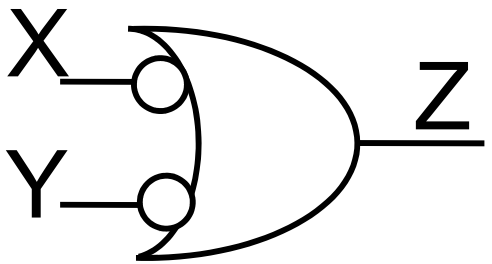
(1) $\overline{X+Y} = \overline{X} \cdot \overline{Y}$





(2) $\overline{X \cdot Y} = \overline{X} + \overline{Y}$





Truth Tables for DeMorgan's

(1) $\overline{X+Y} = \overline{X} \cdot \overline{Y}$

A) X	Y	X + Y	$\overline{X+Y}$	B) X	Y	$\overline{X}$	$\overline{Y}$	$\overline{X} \cdot \overline{Y}$
0	0	0	1	0	0	1	1	1
0	1	1	0	0	1	1	0	0
1	0	1	0	1	0	0	1	0
1	1	1	0	1	1	0	0	0

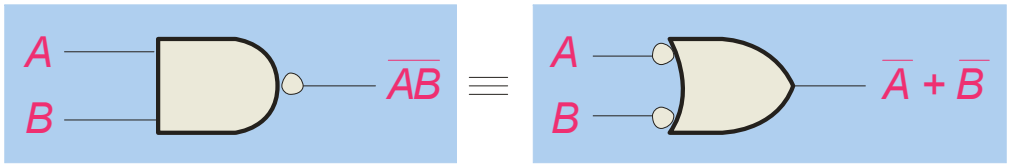
(2) You can try to construct truth table for the other DeMorgan's Theorem.

DeMorgan's Theorem

- The complement of a product of variables is equal to the sum of the complemented variables

Theorem 1

$$\overline{AB} = \overline{A} + \overline{B}$$



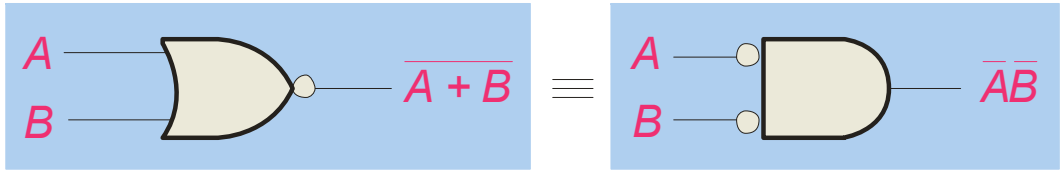
NAND Negative-OR

A	B	$\overline{A \cdot B}$	$\overline{A}+\overline{B}$
0	0	1	1
0	1	1	1
1	0	1	1
1	1	0	0

DeMorgan's Theorem

Theorem 2

$$\overline{A + B} = \overline{A} \cdot \overline{B}$$

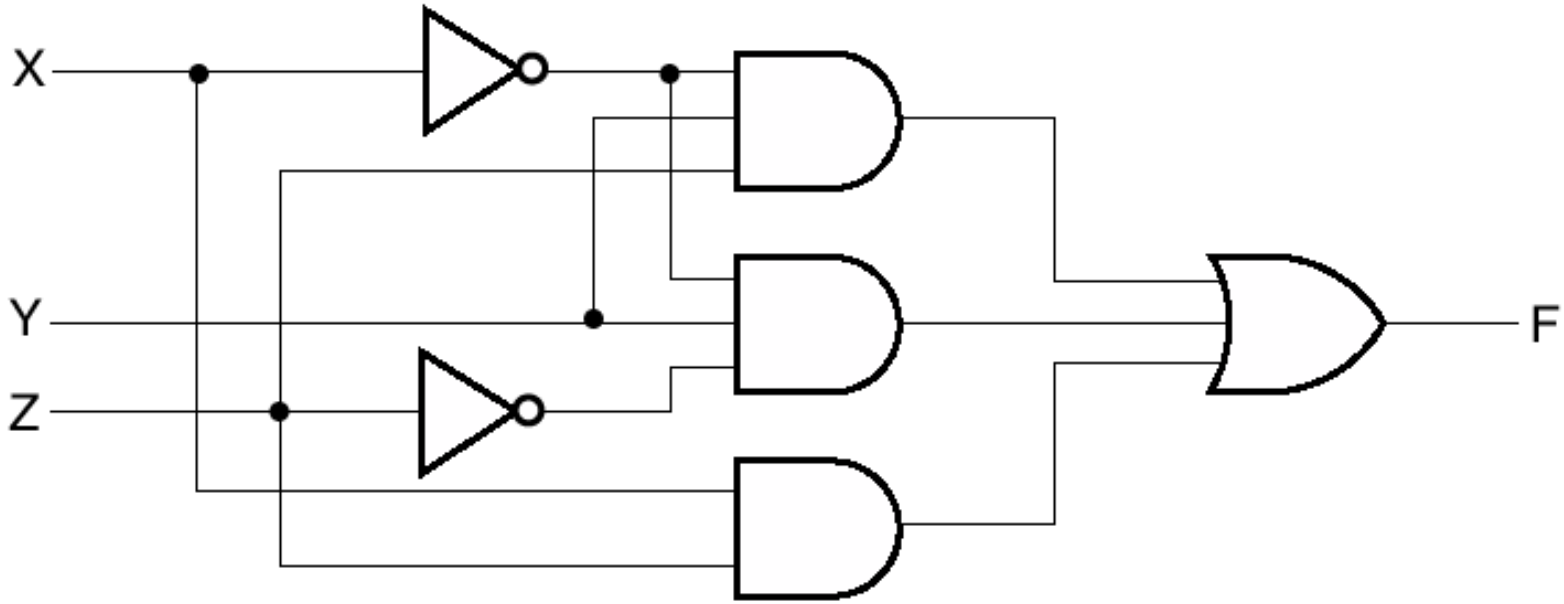


NOR Negative-AND

A	B	$\overline{A+B}$	$\overline{A} \cdot \overline{B}$
0	0	1	1
0	1	0	0
1	0	0	0
1	1	0	0

Algebraic Manipulation

- Consider the following function



$$F = \overline{X}YZ + \overline{X}Y\overline{Z} + XZ$$

Simplify Function

$$F = \overline{X}YZ + \overline{X}Y\overline{Z} + XZ$$

Apply $X(Y+Z) = XY + XZ$

$$F = \overline{X}Y(Z + \overline{Z}) + XZ$$

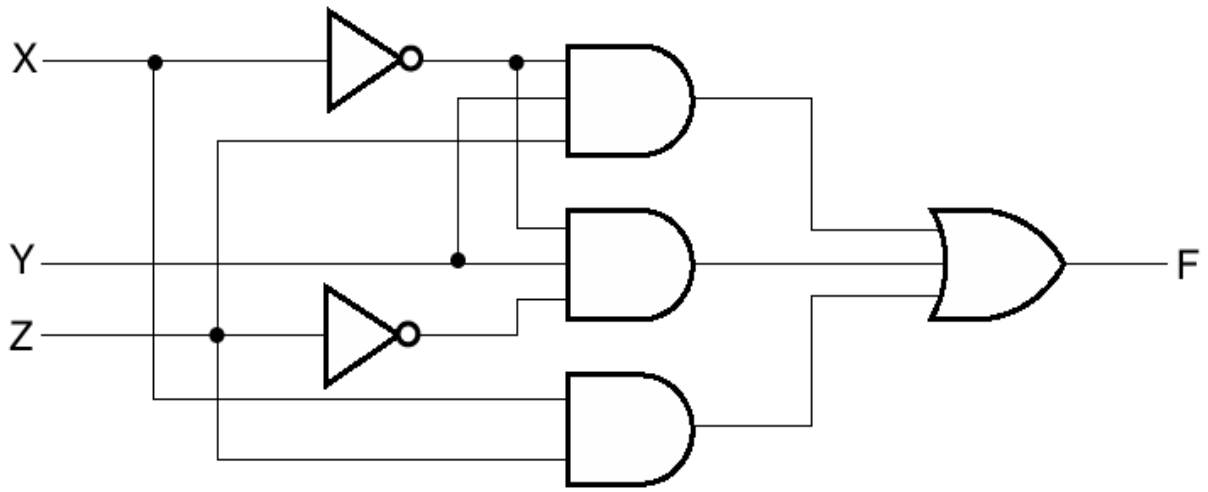
Apply $X + \overline{X} = 1$

$$F = \overline{X}Y \bullet 1 + XZ$$

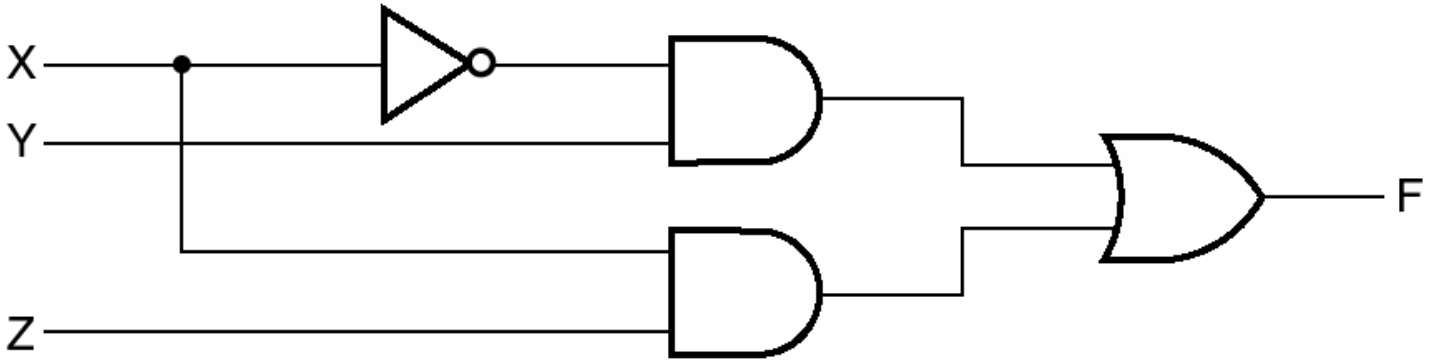
Apply $X \cdot 1 = X$

$$F = \overline{X}Y + XZ$$

Fewer Gates



(a) $F = \overline{X}YZ + \overline{X}Y\overline{Z} + XZ$



(b) $F = \overline{X}Y + XZ$

Implementation of Boolean Function with Gates

$$F = \overline{X}Y + XZ$$

Priority in Simplifying Functions

- Parentheses
- NOT
- AND
- OR

Exercises

1. Apply DeMorgan's theorems to each of the following expressions:

   a. $\overline{(A+B+C)D}$

   b. $\overline{ABC+DEF}$

   c. $\overline{A\overline{B}+\overline{C}D+EF}$

Exercises

2. Simplify the following using Boolean algebra techniques:

$a.\quad A\overline{B} + AB$

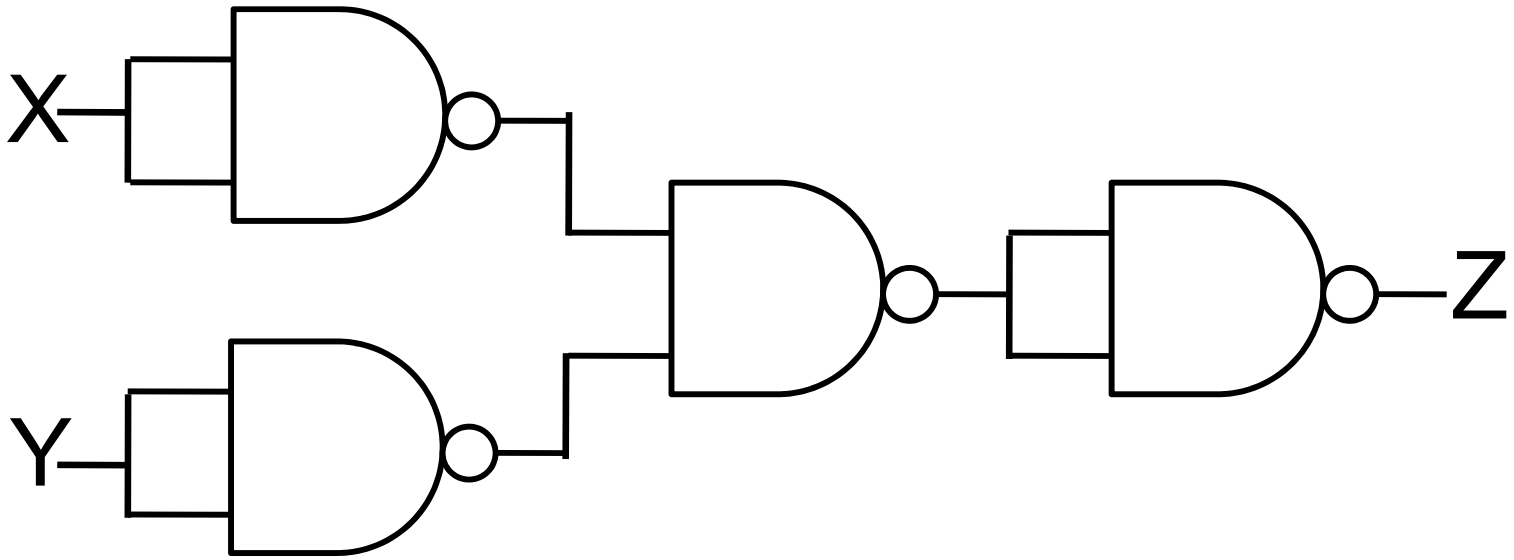
$b.\quad AB + A(B + C) + B(B + C)$

$c.\quad C[\overline{A}\,\overline{B} + A\overline{B}(C + BD)]$

$d.\quad \overline{AB + AC} + \overline{A}\,\overline{B}C$

Exercises

3. Simplify the following circuit.

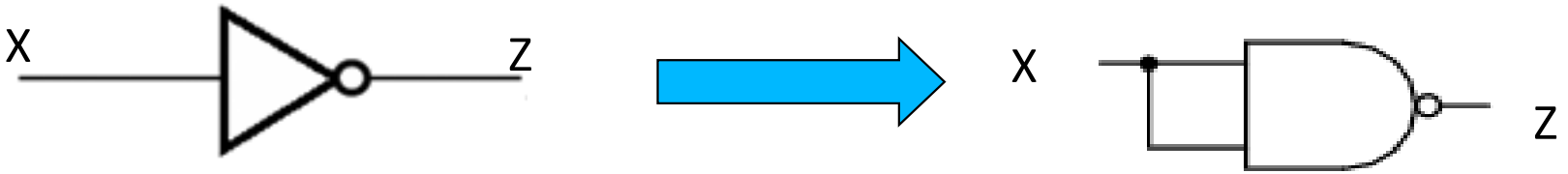


Universal Gates

- NAND and NOR gates are known as Universal gates because all logic gates can be represented by NAND and NOR
- NAND and NOR are the cheapest and smallest to manufacture in Integrated Circuits compared to AND and OR
- Therefore NAND and NOR are almost always used in practical circuit design

NAND Universal Gates

- How to represent inverter using NAND gates?



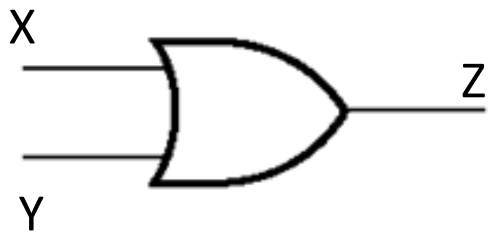
NAND gate truth table

X	Y	$Z=\overline{X\cdot Y}$
0	0	1
~~0~~	~~1~~	~~1~~
~~1~~	~~0~~	~~1~~
1	1	0

Short the inputs together

NAND Universal Gates

- How to represent OR gate using NAND gates?

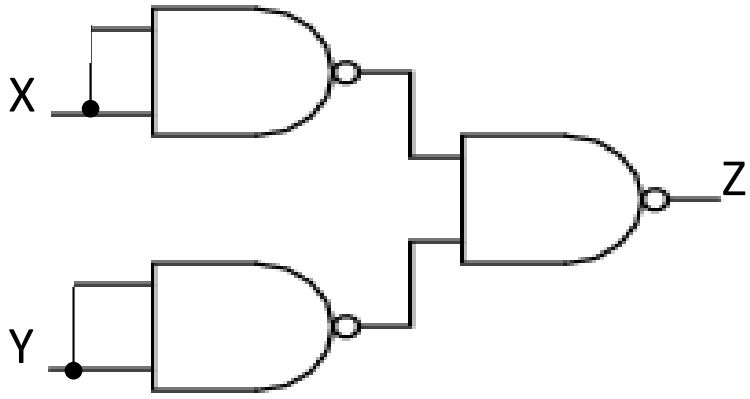


Logic expression for OR gate:

$$X + Y \equiv \overline{\overline{X + Y}}$$

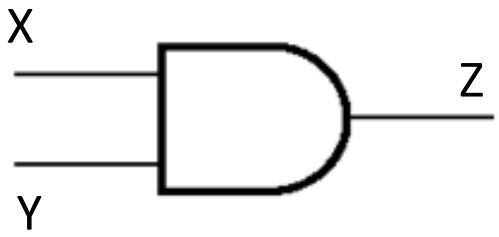
Using DeMorgan's Theorem,

$$\overline{\overline{X} \cdot \overline{Y}}$$



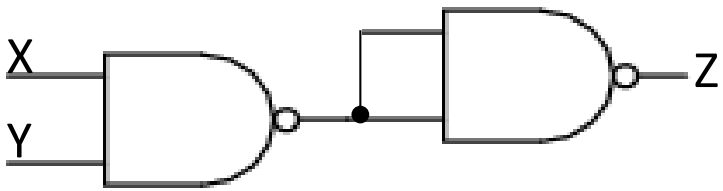
NAND Universal Gates

- How to represent AND gate using NAND gates?



Logic expression for AND gate:

$$X \cdot Y \equiv \overline{\overline{X \cdot Y}}$$



NOR Universal Gates