SKEE1223: Digital Electronics

1 – Number Systems

Standard Systems

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Number Systems

- Standard number systems
  - Decimal
  - Binary
  - Hexadecimal
  - Octal
- Binary Codes
  - BCD 8421
  - Gray Codes
  - ASCII
  - ECBDIC

Binary Numbers

- Counting in binary and decimal:

Binary		Decimal
0 0 0 0	=>	0
0 0 0 1	=>	1
0 0 1 0	=>	2
0 0 1 1	=>	3
0 1 0 0	=>	4
0 1 0 1	=>	5
0 1 1 0	=>	6
0 1 1 1	=>	7
1 0 0 0	=>	8
1 0 0 1	=>	9
1 0 1 0	=>	10
1 0 1 1	=>	11
1 1 0 0	=>	12
1 1 0 1	=>	13
1 1 1 0	=>	14
1 1 1 1	=>	15

How to represent 16?
=> 10000_2

How to represent an arbitrary value, 33?
=> 100001_2

What is the value of 100101_2
=> 37

Binary Numbers (cont.)

- Binary number system uses “0” and “1”
- Example: 5-bit binary:

Bit Position:	4	3	2	1	0	
Binary:	0	0	1	0	1	
Decimal:	0×2^4	0×2^3	1×2^2	0×2^1	1×2^0	
	0 +	0 +	4 +	0 +	1	= 5

Therefore, $00101_2 = 5_{10}$

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Binary Numbers (cont.)

- Convert these binary numbers to decimal:
  - 1010_2 => $2^3 + 2^1 = 10$
  - 10111_2 => $2^4 + 2^2 + 2^1 + 2^0 = 23$
- Convert these decimal numbers to binary:
  - 19 => $2^4 + 2^1 + 2^0 = 10011_2$
  - 58 => $2^5 + 2^4 + 2^3 + 2^1 = 111010_2$

BINARY number → **DECIMAL** number

Method 1: Sum-of-Weights

Binary ↓ Decimal

Given 11001_2

S1: write the weights

weights	2^4	2^3	2^2	2^1	2^0
Binary number	1	1	0	0	1

S2: write the sum of the products of each digit with its weight

$$11001_2 = (1\times2^4)+(1\times2^3)+(0\times2^2)+(0\times2^1)+(1\times2^0)$$
$$= 16 + 8 + 1$$
$$= 25_{10}$$

Answer: 25_{10}

Binary ↑ Decimal ?

Decimal ⇆ Binary Conversion

- Decimal number → binary number
  - Method: sum-of-weights
- Decimal whole number → binary number
  - Method: division-by-2
- Decimal fraction → binary number
  - Method: multiplication-by-2

DECIMAL Number → **BINARY** Number

Method 1: Sum-of-Weights

Decimal

Given 25_{10}

Step 1: Find the power of two that fulfills the following:

a. nearest to the given decimal number; and

b. its decimal number is less than or equal to the given decimal number.

2^2? No, because it is not the nearest. 8 (2^3) is nearer and still less than 25.
2^5? No, because its decimal number, 32 is more than 25.
2^4? Yes, because it is the nearest and its decimal, 16 is less than 25.

Step 2: Subtract the power of two (from Step 1) from the given decimal number.

25-16=9
The result of the subtraction is 9.

Step 3: If the result of the subtraction in Step 2 is 0, go to Step 4. Else, repeat Steps 1 and 2 for the result of the subtraction in Step 2.

Step 4: Write out the binary number based on all the powers of two from Step 1.

Binary

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DECIMAL Number → **BINARY** Number

Method 1: Sum-of-Weights

Given 51_{10}

- Step 1: the power of two which is nearest to 51 but less than 51 is 2^5
- Step 2: the result of subtraction 51-32=19
- Step 3:
  - Repeat Step 1: the power of two nearest to 19 but less than 19 is 2^4
  - Repeat Step 2: the result of subtraction 19-16=3
  - Repeat Step 1: the power of two which is nearest to 3 and less than 3 is 2^1
  - Repeat Step 2: the result of subtraction 3-2=1
  - Repeat Step 1: the power of two which is nearest to 1 and equal to 1is 2^0
  - Repeat Step 2: the result of subtraction 1-1=0
- Step 4:

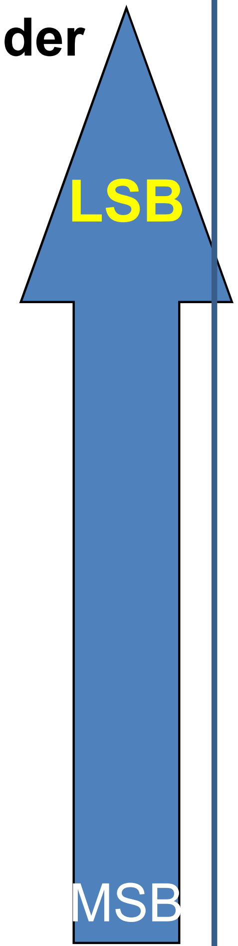
Weights	2^5	2^4	2^3	2^2	2^1	2^0
Binary number	1	1	0	0	1	1

DECIMAL Number → **BINARY** Number

Method 2: Repeated Division-by-2

- Method 1 can convert both **whole** numbers and **fractional** numbers to binary.
- Method 2 is to convert **whole** numbers to binary.
- Given 45_{10}
  - Repeat the division until the **quotient** is 0.

Quotient	Remainder
$\frac{45}{2} = 22$	1
$\frac{22}{2} = 11$	0
$\frac{11}{2} = 5$	1
$\frac{5}{2} = 2$	1
$\frac{2}{2} = 1$	0
$\frac{1}{2} = 0$	1



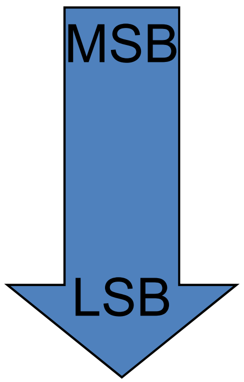
The binary number is 101101_2

DECIMAL Fraction → **BINARY Fraction**

Method 3: Repeated Multiplication-by-2

- Method 3 is to convert decimal fraction to binary.
- Repeat the multiplication until the fractional part is all zeros.
- Given 0.3125

					Carry
0.3125	x	2	=	0.625	0
0.625	x	2	=	1.25	1
0.25	x	2	=	0.50	0
0.50	x	2	=	1.00	1



The binary fraction is .0101

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Includes 9 Chapter
Last chapter on Programmable Logic Device
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Hexadecimal System

- Base-16 system
- 16 symbols: 10 numeric digits and 6 alphabetic characters
  - 0,1,2,3,4,5,6,7,8,9
  - A,B,C,D,E,F
- Compact way of writing binary number
- Widely used in computer and microprocessor applications

Hexadecimal Numbers

- Counting in hexadecimal

Binary		Decimal		Hexadecimal
0 0 0 0	=>	0	=>	0
0 0 0 1	=>	1	=>	1
0 0 1 0	=>	2	=>	2
0 0 1 1	=>	3	=>	3
0 1 0 0	=>	4	=>	4
0 1 0 1	=>	5	=>	5
0 1 1 0	=>	6	=>	6
0 1 1 1	=>	7	=>	7
1 0 0 0	=>	8	=>	8
1 0 0 1	=>	9	=>	9
1 0 1 0	=>	10	=>	A
1 0 1 1	=>	11	=>	B
1 1 0 0	=>	12	=>	C
1 1 0 1	=>	13	=>	D
1 1 1 0	=>	14	=>	E
1 1 1 1	=>	15	=>	F

How to represent 16 in hexadecimal?

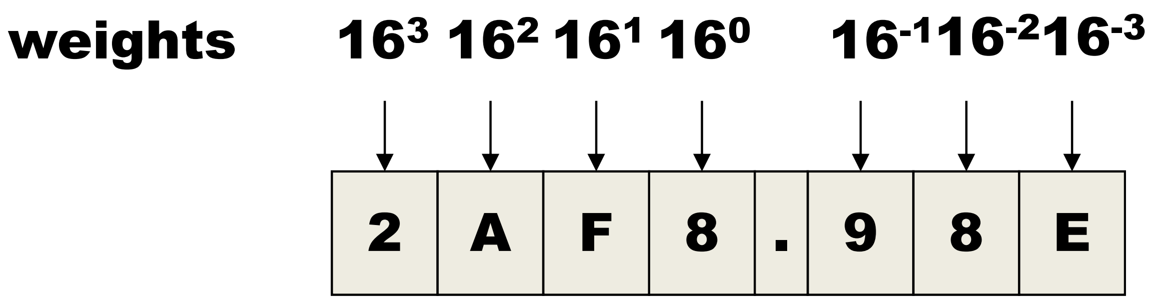
=> 10_{16}

Continue counting..

11, 12, 13, 14, 15, 16, 17, 18, 19, 1A
1B, 1C, 1D, 1E, 1F, 20…

Hexadecimal System

- Examples: $1C_{16}$, $A85_{16}$
- 1CH, A85H
- The position of each digit in a hexadecimal number can be assigned a weight
- For example: 2AF8.98E
  - 2AF8.98E is a **hexadecimal number**
  - 2 is a digit, A is a digit, F is a digit…



Hexadecimal Counting

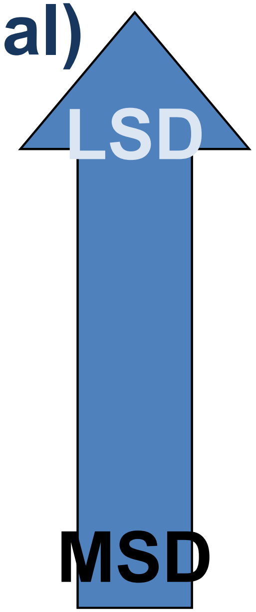
- 0→1→2→3→4→5→6→7→8→9→A→B→C →D→E→F→10→11→12-------------------- →19→1A--- →FF→100→101→102-------------------

DECIMAL Number → **HEXADECIMAL** Number

Method: Repeated division-by-16

- Repeat the division until the quotient is 0.
- Example: 650_{10}

quotient	Remainder (decimal)	Remainder (hexadecimal)
$\frac{650}{16} = 40$	10	A
$\frac{40}{16} = 2$	8	8
$\frac{2}{16} = 0$	2	2



So, the hexadecimal number is $28A_{16}$

BINARY Number → **HEXADECIMAL** Number

- Step 1: Break the binary number into 4-bit groups, starting from LSB.
- Step 2: Replace each 4-bit with the equivalent hexadecimal number.
- Example: 111111000101101001

binary	0011	1111	0001	0110	1001
hexadecimal	3	F	1	6	9

The hexadecimal number is $3F169_{16}$

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HEXADECIMAL Number → **BINARY** Number

- Step: Replace each digit of the hexadecimal number with the equivalent 4-bit binary number.
- Example: $CF8E_{16}$

Hexadecimal	C	F	8	E
Binary	1100	1111	1000	1110

The binary number is 1100111110001110_2

Hexadecimal Numbers (cont.)

- Hexadecimal number conversion

break binary into 4 groups

Binary: 0 0 0 1 | 0 1 1 0 | 1 1 0 1 | 1 0 0 1

1 | 6 | D | 9

Hexadecimal: 1 6 D 9 $_{16}$

Can you convert this hex number to decimal?

Hexadecimal Numbers (cont.)

- Convert the following to binary:
  - $CF8E_{16}$ => $1100\ 1111\ 1000\ 1110_2$
  - 974_{16} => $1001\ 0111\ 0100_2$
- Convert the following to hexadecimal
  - $1111\ 0000\ 1010_2$ => $F0A_{16}$
  - $10\ 0001\ 1101\ 1001_2$ => $21D9_{16}$

Octal System

- Base-8 system
- 8 digits: 0,1,2,3,4,5,6,7
- Convenient way to express binary numbers and codes. Use 3 bits of binary

Octal Numbers

- Counting in Octal

Binary		Decimal		Hexadecimal		Octal
0 0 0 0	=>	0	=>	0	=>	0
0 0 0 1	=>	1	=>	1	=>	1
0 0 1 0	=>	2	=>	2	=>	2
0 0 1 1	=>	3	=>	3	=>	3
0 1 0 0	=>	4	=>	4	=>	4
0 1 0 1	=>	5	=>	5	=>	5
0 1 1 0	=>	6	=>	6	=>	6
0 1 1 1	=>	7	=>	7	=>	7
1 0 0 0	=>	8	=>	8	=>	10
1 0 0 1	=>	9	=>	9	=>	11
1 0 1 0	=>	10	=>	A	=>	12
1 0 1 1	=>	11	=>	B	=>	13
1 1 0 0	=>	12	=>	C	=>	14
1 1 0 1	=>	13	=>	D	=>	15
1 1 1 0	=>	14	=>	E	=>	16
1 1 1 1	=>	15	=>	F	=>	17

BINARY Number → **OCTAL** Number

- Step 1: Break the binary number into 3-bit groups, starting from LSD.
- Step 2: Replace each 3-bit group with the equivalent octal number.
- Example: 110101_2

binary	110	101
octal	6	5

The octal number is 65_8

OCTAL Number → **BINARY** Number

- Step: Replace each octal digit with the equivalent 3-bit group.
- Example: 13_8

Octal	1	3
Binary	001	011

So, the binary number is $\mathbf{001011_2}$

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Octal Numbers (cont.)

- Octal numbers conversion

break binary into 3 groups

Binary: 1 0 1 1 1 1 1 0 1 0 0 0 1

1 3 7 2 1

Octal: $1\,3\,7\,2\,1_8$

Can you convert this octal number to decimal?

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Octal Numbers (cont.)

- Convert the following to binary
  - 25_8 => $10\ 101_2$
  - 140_8 => $001\ 100\ 000_2$
- Convert the following to octal
  - $110\ 101_2$ => 65_8
  - $1\ 101\ 111\ 001_2$ => 1571_8

Exercise

- Fill in the blanks.

Decimal	Binary	Hexadecimal	Octal
	1101.011_2		
	10101.11_2		
245.625_{10}			
703_{10}			
		$A85_{16}$	

Exercise

- *Answers*

Decimal	Binary	Hexadecimal	Octal
13.375_{10}	1101.011_2	$D.6_{16}$	15.3_8
21.75_{10}	10101.11_2	$15.C_{16}$	25.6_8
245.625_{10}	11110101.101_2	$F5.A_{16}$	365.5_8
703_{10}	1010111111_2	$2BF_{16}$	1277_8
2693_{10}	101010000101_2	$A85_{16}$	5205_8

Binary Arithmetic

- Binary addition
- Binary subtraction
- Binary multiplication
- Binary division

Binary addition

- 0 + 0 = 0 with a carry of 0
- 0 + 1 = 1 with a carry of 0
- 1 + 0 = 1 with a carry of 0
- 1 + 1 = 10 with a carry of 1
- Example: 111 + 11 = ?

In decimal…

111	7
+ 11	+3
1010	10

Binary subtraction

- 0 – 0 = 0
- 1 – 1 = 0
- 1 – 0 = 1
- 10 – 1 = 1
- Example: 101 – 11 = ?

	Binary	In decimal
	101	5
-	11	-3
	10	2

Binary Multiplication

- 0 x 0 = 0
- 0 x 1 = 0
- 1 x 0 = 0
- 1 x 1 = 1
- Example: 101 x 111 = ?

```
         101          5
x        111         x7
         101         35
        101
       101
     100011
```

Binary Division

- The procedure is same as decimal division
- Examples: 110 ÷ 11 =?

```
      10
    ______
11 ) 110
     11
     ___
       0
       0
```

In decimal…

```
      2
    ______
3  )  6
      6
     ___
      0
```

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Review

- Why binary numbers? Why not decimal?
  - Digital system understand binary numbers, not decimal numbers.